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Holomorphic Endomorphisms of the Unit Disk with Invariant Diameter and Bounded Distortion

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Abstract—We consider the problem of embeddability of a holomorphic endomorphism of the unit disk with invariant diameter and bounded distortion into a one-parameter semigroup; it is assumed that elements of the semigroup have the same properties as the original map does. We obtain embeddability criteria formulated in terms of the Königs function.

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1. INTRODUCTION

In the paper we continue investigations of embeddability of a holomorphic mapping into a one-parameter semigroup. The problem arises in areas of natural sciences that use dynamics of holomorphic mappings for description the investigated processes. In the theory of analytic functions it has appeared first as the problem of fractional integration. The latter consists in finding (if exists) of a family of functions f^t, $t \geqslant 0$, satisfying

$$f^0(z) = z, \;\; f^1(z) = f(z) \;\text{ and }\; f^{t+s}(z) = f^t \circ f^s(z) \;\text{ for all }\; s \geqslant 0, \;\; t \geqslant 0;$$

here f is a given function.

Now we describe briefly some cases in investigation the problem of fractional integration. The first research was made in the papers by Schröder [1] and Königs [2]. It concerns the local case when f and its iterations are differentiable, in complex sense, in neighborhoods of a common fixed point; every iteration has its own neighborhood. Schröder connected the problem of fractional integration with solvability of a functional equation. Königs suggested a construction giving a solution to the equation and has shown the existence of fractional iterations f^t, $t \geqslant 0$, under non-essential restrictions on f.

Further the problem of fractional integration was investigated for entire and meromorphic functions. The obtained results are opposite to the local cases. Thus, in [3, 4] it was shown that existence of fractional iterations is possible only for Möbius functions.

Further an investigation was developed of analytic in a plane domain functions which attain values from the same domain; as a rule, the domain coincides either with the unit disk or with the half-plane. The development of this direction was essentially influenced by [5–7]. The direction, in contrast to the first ones described above, is more various in the sense of obtained results, and its active development is stimulated by a wide range of applications in the theory of branched stochastic processes, the non-commutative probability theory, the theory of composite operators, etc. Importance of this direction and increased recent interest to it are connected with new applications of the Löwner equation such as SLE (Shramm (stochastic) – Löwner evolution); they allowed to solve some difficult problems in various fields of natural sciences.

Essential progress in investigation of the problem of fractional integration was made when they began to consider families of fractional iterations as one-parametric semigroups. As in general theory

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