ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 3, pp. 45–62. © Allerton Press, Inc., 2010.
Original Russian Text © N. Temirgaliev, S.S. Kudaibergenov, and A.A. Shomanova, 2010, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2010, No. 3, pp. 52–71.

Applications of Smolyak Quadrature Formulas to the Numerical Integration of Fourier Coefficients and in Function Recovery Problems

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Received November 16, 2007; in final form, May 4, 2009

Abstract—In this paper with the help of Smolyak quadrature formulas we calculate exact orders of errors of the numerical integration of trigonometric Fourier coefficients of functions from generalized classes of Korobov and Sobolev types. We apply the obtained results to the recovery of functions from their values at a finite number of points in terms of the K. Sherniyazov approach.

DOI: 10.3103/S1066369X10030084

Key words and phrases: *Ul'yanov classes, numerical integration, Smolyak quadrature formula, trigonometric Fourier coefficients, recovery operators.*

Dedicated to the 80th anniversary of the RAS academician P. L. Ul'yanov

1. INTRODUCTION

This paper is dedicated to concretizations (connected with the Smolyak grid) of the following general recovery problem (see [1] and references therein).

Let normed spaces X and Y of numerical functions defined on sets Ω and Ω_1, respectively, be given. Let $F \subset X$ and let a mapping $Tf = u(y, f)$ act from F to Y.

For each integer $N \geq 1$ we denote by $\{(l^{(N)}; \varphi_N)\}$ the set of all pairs $(l^{(N)}; \varphi_N)$ that consist of a collection of N functionals $l^{(N)} = (l_1, \dots, l_N)$, $l_j(\cdot) : F \mapsto C$, $j = 1, \dots, N$ (when speaking of the linearity of l_j we mean the linearity on the linear shell of F) and a function

$$\varphi_N(\tau_1, \dots, \tau_N; y) : C^N \times \Omega_1 \mapsto C;$$

assume also that $D_N \subset \{(l^{(N)}; \varphi_N)\}$.

The problem is to obtain the upper and lower estimates (it is desirable that they should coincide accurate to a constant) for the value

$$\delta_N \equiv \delta_N(D_N; T, F)_Y = \inf_{(l^{(N)}; \varphi_N) \in D_N} \sup_{f \in F} \|u(\cdot; f) - \varphi_N(l_1(f), \dots, l_N(f); \cdot)\|_Y, \tag{1}$$

i.e., a computer (computational) diameter, and to find a pair $(l^{(N)}; \varphi_N)$ in D_N that realizes the upper bound.

By concretizing spaces X and Y, classes F ($F \subset X$), operators T, and sets D_N in formula (1) we obtain various statements of the problem.

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