

The Rate of Polygonal Approximation of a Non-Rectifiable Curve and the Jump Problem

B. A. Kats^{1*}

¹Kazan State Architecture and Building University, ul. Zelyonaya 1, Kazan, 420043 Russia

Received March 3, 2008

Abstract—We consider certain boundary value problems for functions holomorphic in domains bounded by closed non-rectifiable curves in the complex plane. We study the solvability of these problems in dependence of the rate of the polygonal approximation of the mentioned curves.

DOI: 10.3103/S1066369X1005004X

Key words and phrases: *holomorphic functions, jump problem, non-rectifiable curves*.

1. INTRODUCTION

Let Γ be a simple Jordan closed curve on the complex plane \mathbf{C} dividing it into two domains: a finite domain D^+ and a domain D^- containing the infinity point. A function $f(t)$ is defined on this curve and satisfies there the Hölder condition

$$\sup \left\{ \frac{|f(t') - f(t'')|}{|t' - t''|^\nu} : t', t'' \in \Gamma, t' \neq t'' \right\} := h_\nu(f, \Gamma) < \infty \quad (1)$$

with some exponent $\nu \in (0, 1]$; in what follows $H_\nu(\Gamma)$ stands for the set of all functions defined on Γ that satisfy inequality (1). Consider the boundary value problem on the evaluation of a holomorphic in $\overline{\mathbf{C}} \setminus \Gamma$ function $\Phi(z)$ such that it has limit values $\Phi^+(t)$ and $\Phi^-(t)$ for z tending to any point $t \in \Gamma$ from D^+ and D^- , correspondingly; the limits have to satisfy the boundary value condition

$$\Phi^+(t) - \Phi^-(t) = f(t), \quad t \in \Gamma; \quad (2)$$

in addition, we assume that $\Phi(\infty) = 0$.

This boundary value problem is well known as the jump problem (see, e.g., [1], P. 106). Its studies have more than centennial history related to researches of the Cauchy type integral

$$\Phi(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{f(\zeta) d\zeta}{\zeta - z}. \quad (3)$$

Even Harnak and Morera were aware (in a varying degree) that a Cauchy type integral has continuous boundary values on Γ from both sides if the density $f(t)$ satisfies the Hölder condition with any exponent ν from the interval mentioned above and the path of integration is piecewise smooth. Sokhotskii and Plemelj proved that the difference of these boundary values equals the density of the integral ([1], P. 86). Since then restrictions on the curve Γ were weakened several times. For instance, E. M. Dyn'kin [2] and T. Salimov [3] proved independently that under the assumption

$$\nu > \frac{1}{2} \quad (4)$$

integral (3) with the density $f \in H_\nu(\Gamma)$ over a non-smooth rectifiable closed curve Γ has boundary values $\Phi^\pm(t)$ satisfying the Sokhotskii–Plemelj formula, and restriction (4) cannot be improved on the class of all rectifiable curves. Then the author studied the solvability of the jump problem on non-rectifiable

*E-mail: katsboris877@gmail.com.