ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 5, pp. 6–14. © Allerton Press, Inc., 2010.
Original Russian Text © V.A. Emelichev, A.V. Karpuk, and K.G. Kuz'min, 2010, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2010, No. 5, pp. 8–17.

On a Measure of Quasistability of a Certain Vector Linearly Combinatorial Boolean Problem

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Received March 31, 2008

Abstract—We consider a multicriteria problem of finding the Pareto set in the case when linear forms (functions) are minimized both on a set of permutations and on a set of Boolean vectors. We obtain a formula for the radius of that type of the problem stability (with respect to perturbations of parameters of the vector criterion) that guarantees the preservation of all Pareto optimal solutions of the initial problem and allows the occurrence of new ones.

DOI: 10.3103/S1066369X10050026

Key words and phrases: *linearly combinatorial Boolean problem, vector objective function, Pareto set, quasistability radius, perturbing matrix.*

1. INTRODUCTION

Multicriteria combinatorial decision-making models often occur in management, projecting, economics, and many other realms of applied research. That is why the interest of mathematicians to vector (multicriteria) problems of discrete optimization does not fade away; this is confirmed by new publications in this area (see, e.g., references in [1]). One of the most known approaches to the study of stability of discrete optimization problems (both scalar and vector ones) is a constructive technique for obtaining qualitative stability characteristics of the set of optimal solutions. Mostly often, as these characteristics one uses radii of various types of stability defined as the limit level of perturbations of the problem parameters that keep some (given in advance) property of the set of its optimal solutions. The perturbed parameters, as a rule, are coefficients of a scalar or vector criterion. When studying the stability radii, one obtains their formal expressions or algorithms for their calculation. In the case of a scalar criterion one has also obtained formulas for the stability radii for the Boolean programming problems, problems on systems of subsets and on graphs [2], as well as for some problems in the scheduling theory [3]. In the case of a vector criterion, for certain discrete optimization problems one has succeeded in obtaining analogous results (e.g., [4–8]). Not going to dwell on the description of the whole spectrum of related issues, we refer the reader to ample references in [9].

In this paper we consider a vector problem, where linear functions are minimized on the aggregate of two finite sets. We obtain a formula for the radius of quasistability, i.e., the stability of a problem to perturbations of parameters of its vector criterion that keeps the Pareto optimality of all solutions. We adduce several corollaries. In particular, we give a sufficient condition (in the linear case with the only permutation it is also necessary) for the problem quasistability. Note that some of these results were announced in [10].

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