

# The Upper Semicontinuity of Senior Generalized Lyapunov Exponents of Systems of Differential Equations

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**Abstract**—We study the generalized Lyapunov exponents, i.e., the Lyapunov exponents in a more general scale, and apply them for studying the asymptotics of the growth of solutions to differential systems. We obtain necessary and sufficient conditions for the upper semicontinuity of the senior generalized Lyapunov exponents in a class of systems of differential equations.

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In this paper we establish necessary and sufficient conditions for the upper semicontinuity of the senior generalized Lyapunov exponent in a class of systems of differential equations.

In linear systems of differential equations with continuous and unbounded coefficients, exponents may not have a finite value. This fact disables us to use results (see [1], P. 10) of the theory of Lyapunov exponents in such cases, because even the initial definition turns out to be not completely applicable. If exponents of a system equal zero, then difficulties arise, because in this case we say nothing essential about the behavior of solutions as  $t \rightarrow \infty$ . Therefore we consider generalized Lyapunov exponents, i.e., Lyapunov exponents in a more general scale, and use them for studying the asymptotics of the growth of solutions to differential systems.

Let  $Q$  be the set of all piecewise continuous, nonnegative, unbounded, monotonically increasing functions with  $t \geq 0$ .

The generalized Lyapunov exponent of a nonzero solution  $x(t)$  to a system with a piecewise continuous matrix of coefficients  $A(t)$ ,  $t \in I \equiv [0, +\infty]$ ,

$$\dot{x} = A(t)x \quad (1)$$

with respect to  $q(t)$ ,  $t \in I$ , is defined by the formula

$$\chi[x, q] = \overline{\lim}_{t \rightarrow +\infty} \frac{1}{q(t)} \ln \|x(t)\|.$$

Depending on  $q(t)$ , the generalized exponent is a number or a symbol  $-\infty$  or  $+\infty$ . For the null solution  $x(t) = \theta$  by definition we put  $\chi[\theta, q] = -\infty$ . It is known (see [2]) that if a vector function  $F(t, x)$  is continuous in a domain  $G = I \times D$ ,  $D \subset R^n$ ,  $F(t, 0) = 0$  and satisfies the condition  $\|F(t, x)\| \leq K\varphi(t)\|x\|$ , where  $\varphi(t)$  is a continuous positive in  $I$  function and  $K$  is a positive constant, then all nonzero solutions of the vector equation  $\dot{x} = F(t, x)$  have finite generalized exponents with respect to

$q(t) = \int_{t_0}^t \varphi(\tau) d\tau$ ,  $t_0 \in I$ . In particular, this is true for system (1) subject to  $\|A(t)\| \leq K\varphi(t)$ . Therefore system (1) has no more than  $n \in N$  different finite generalized exponents with respect to  $q(t) = \int_{t_0}^t \varphi(\tau) d\tau$ , because there exist only  $n$  linearly independent solutions. The greatest generalized exponent of system (1) is called the senior generalized exponent of system (1) and is denoted by  $\lambda(A)$ .

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