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ON A CLASS OF TWO-LAYER DIFFERENCE SCHEMES FOR NONLINEAR BOUNDARY VALUE PROBLEMS WITH MEMORY

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The two-layer difference schemes (DS) are widely used in solving nonstationary problems of Mathematical Physics. Various aspects of the theory of these schemes are studied in a number of papers. DS for the linear problems are studied most completely. In [1]–[4] a general theory of well-posedness was constructed for linear two-layer operator-difference schemes (ODS), which allows to study from the same standpoint the solvability, stability, and convergence of the DS for evolution equations and systems.

At the same time, the well-posedness theory for nonlinear DS seems to be developed essentially weaker. We should note [5]–[14], where the well-posedness theorems were proved for certain types of nonlinear two-layer DS for parabolic equations. In [15]–[16], the well-posedness of two-layer nonlinear ODS was studied, where the "spatial" operator was presented as a superposition of a pair of operators with different properties. This representation makes it possible to state the well-posedness conditions for a sufficiently large class of DS for both parabolic and hyperbolic equations. In the present article we shall study the well-posedness of a system of operator equations, whose particular case is the ODS considered in [15]. DS of that kind can be used to solve a wide class of problems with "memory".

Proved in the article for a system of operator equations, the well-posedness theorem is then applied to investigation of the solvability and convergence of DS for a system of the two-dimensional hydrodynamic equations in the Lagrangian coordinates.

1. On well-posedness of the two-layer ODS

Let $H = H_h = H_{1,h} \otimes H_{2,h}$ be a family of finite-dimensional Euclidean spaces which depend on the parameter h belonging to a finite-dimensional space with a norm $|h| > 0$, $\overline{\omega}_\tau = \{0, \tau, 2\tau, \dots, T\}$ be a grid on the segment $[0, T]$, $X = X_{\tau h} = \{(v(t), \chi(t)) \in H,\ t \in \overline{\omega}_\tau\}$ be a space of vector-valued functions defined on $\overline{\omega}_\tau$ with the range in H.

Consider an ODS of the form

$$\begin{cases} y_t + A(\chi^{(\sigma)}, \tilde{D}(y, \hat{y})) = \varphi(t), \\ \chi_t = P(\chi^{(\sigma)}, \tilde{D}(y, \hat{y})), \end{cases} \quad t \in \omega_\tau = \overline{\omega}_\tau \backslash \{T\}, \quad y(0) = y_0, \quad \chi(0) = x_0, \tag{1}$$

where $\sigma \in [0, 1]$ is a numerical parameter, the layer's weight, $v^{(\sigma)} = \sigma\hat{v} + (1 - \sigma)v$, $\hat{v}(t) = v(t + \tau)$, $\tilde{D}(y, v) = \int_0^1 D(\sigma\hat{v} + (1 - \sigma)y)d\sigma$, $(y, \chi) \in X$, $(y_0, x_0) \in H$, D is a nonlinear operator from H_1 to H_1, A and P are nonlinear operators acting in H with the ranges in H_1 and H_2, respectively.

A particular case of ODS (1) are the ODS studied in [12], [15], [16].

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