The Mobility of Riemannian Spaces With Respect to Conformal Mappings Onto Einstein Spaces

L. E. Evtushik[1*], V. A. Kiosak[2**], and J. Mikeš[3***]

[1] *Moscow State University, Vorob'yovy Gory 1, Moscow, 119991 Russia*

[2] *Danube Branch of Interregional Human Resource Management Academy, ul. Klushina 3a, Izmail, Odessa Region, 68607 Ukraine*

[3] *Palacky University, ul. 17 Listopadu 12, Olomouc, 771 46 Czech Republic*

Abstract—We give an estimate of the first lacuna in the distribution of mobility degrees r of n-dimensional (pseudo-)Riemannian spaces with respect to conformal mappings onto Einstein spaces. We obtain a tensor characteristic of spaces which are not conformally flat and have $r = n - 1$, which is the maximum possible value. Thus, we have found maximum mobile non-conformally flat spaces with $r = n - 1$.

1. INTRODUCTION

Conformal mappings of n-dimensional (pseudo-)Riemannian spaces V_n were studied by many authors. Note that conformal mappings have applications in the general theory of relativity (see, e.g., [1−5]).

The question of whether a space V_n $(n > 2)$ admits a conformal mapping onto an Einstein space $\overline{V}_n$ has been reduced by G. Brinkmann [6] to the problem of existence of a solution of a certain nonlinear system of differential equations in covariant derivatives of Cauchy type with respect to $(n + 1)$ unknown functions. This problem is described in detail in A. Z. Petrov's monograph [2].

In [7]([8, 9]) the main system has been reduced to a linear system the use of which made it possible to estimate the number r of parameters in the solution of the above indicated problem. We call this number *the degree of mobility of a Riemannian space with respect to conformal mappings onto Einstein spaces*.

In this paper, we estimate the first lacuna in the distribution of mobility degrees of Riemannian spaces with respect to conformal mappings onto Einstein spaces. As is known [7], conformally flat Riemannian spaces and only these admit the maximum value $r = n + 2$. We obtain a tensor characteristic of non-conformally flat spaces with $r = n - 1$, which is the maximum possible value. Thus, we have estimated the first lacuna in the distribution of mobility degrees of Riemannian spaces with respect to conformal mappings onto Einstein spaces and found maximum mobile non-conformally flat spaces with respect to the indicated mappings.

The investigations are of local character. The geometric objects which appear in what follows are assumed to be smooth of sufficiently high order. The dimensions of spaces under consideration is greater than three, $n > 3$.

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[*]E-mail: ngus12@mail.ru.
[**]E-mail: vkiosak@ukr.net.
[***]E-mail: mikes@inf.upol.cz.