ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2008, Vol. 52, No. 7, pp. 8–14. © Allerton Press, Inc., 2008.
Original Russian Text © A.L. Kuz'mina, 2008, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2008, No. 7, pp. 11–18.

$L^p(AP)$ Spaces ($1 \leq p \leq \infty$) and Their Adjoint Ones

A. L. Kuz'mina[1]

[1] *Kazan State University, ul. Kremlyovskaya 18, Kazan, 420008 Russia*

Received May 7, 2004

Abstract—We study Besicovitch-type spaces of generalized almost periodic functions. The main result is a theorem on representation of linear continuous functionals that is similar to the classical result of F. Riesz.

DOI: 10.3103/S1066369X08070025

Key words and phrases: *almost-periodic functions, Besicovitch space, linear continuous functionals.*

In this paper we study L^p spaces, i.e., the spaces of integrable almost periodic (a. p.) functions $L^p(AP)$ $(1 \leq p \leq \infty)$ and spaces adjoint to them.

In Item 1 we introduce spaces of a. p. functions $L^p(AP)$ $(1 \leq p \leq \infty)$ as completions of spaces AP of functions which are a. p. in the sense of Bohr with respect to the norm

$$\|\cdot\|_p = \left(\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |\cdot|^p \, dt\right)^{1/p} \quad (1 \leq p < \infty)$$

and

$$\|\cdot\|_\infty = \lim_{p\to\infty} \|\cdot\|_p.$$

We prove that one can introduce the scalar product for functions of spaces $L^p(AP)$ and $L^q(AP)$ $(1/p + 1/q = 1, 1 \leq p \leq \infty)$ which are mutually adjoined in the sense of Young.

In Item 2 we prove the theorem on the general form of a linear continuous functional on $L^p(AP)$ $(1 \leq p < \infty)$. This theorem implies that spaces which are adjoined in the sense of Young are also adjoint, i.e.,

$$[L^p(AP)]^* = L^q(AP) \ \ (1/p + 1/q = 1, \ \ 1 \leq p < \infty).$$

1. Spaces $L^p(AP)$ $(1 \leq p \leq \infty)$ considered as completions of the space AP with a proper norm.

AP is the linear space of functions $x(t)$, $-\infty < t < \infty$, a. p. in the sense of Bohr ([1], Chap. 1, § 1). If $x(t) \in AP$, then $|x(t)| \in AP$ and if $y(t) \in AP$, then $x(t)y(t) \in AP$. For each function $x(t) \in AP$ one can find the integral average value ([1], § 3)

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)dt \ \ (\neq \infty).$$

Let us prove that functions $|x(t)|^p$ $(0 < p < \infty)$ also belong to AP, if $x(t)$ belongs to AP.

Indeed, the inequality

$$\big||x(t+\tau)|^\alpha - |x(t)|^\alpha\big| \leq \big||x(t+\tau)| - |x(t)|\big|^\alpha \ \ (0 < \alpha < 1)$$

implies that if τ is an ε-almost period of the function $x(t)$, then τ also is an ε^α-almost period of the function $|x(t)|^\alpha \in AP$ $(0 < \alpha < 1)$.

With integer p $(p > 1)$ the function $|x(t)|^p$ belongs to AP, because it is a finite product of functions $|x(t)|$, $x(t) \in AP$. Evidently, with any p, $1 < p < \infty$, the function

$$|x(t)|^p = |x(t)|^{[p]}|x(t)|^\alpha, \quad p = [p] + \alpha, \ \ 0 < \alpha < 1,$$