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THE CARATHEODORY D-CONDITIONS AND THEIR CONNECTION WITH THE D-CONTINUITY OF THE NEMYTSKIĬ OPERATOR

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The Nemytskiĭ operator h (other term — superposition operator) is of importance in nonlinear analysis, being the object of numerous investigations in the second half of this century (see [1]). Its properties are defined by the features of the function $f(t, x)$, $t \in T$, $x \in X$, which generates this operator: $h\varphi(t) = f(t, \varphi(t))$ (T and X are supports of the measurable and topological structures, respectively).

In particular, Caratheodory conditions ("measurability with respect to t and continuity with respect to x") ensure not only the superposition measurability (briefly, supmeasurability) of the function f (in other words, transformation by operator h of the measurable functions φ to measurable $h\varphi$), but also the continuity of the operator h with respect to the measure (see, e.g., [2], § 17). From [3]–[6] (see also surveys [7], [8]) it follows that, in rather general situation, the Caratheodory conditions are equivalent to the so-called "standard property" of the function f and the continuity of the operator h with respect to measure.

In the present article these results are generalized in various directions. First, the function f can be defined for not all $t \in T$, $x \in X$, but only on a certain set $\Omega \subset T \times X$ (on this matter see [6], remark 3), whose properties play certain role in discussed questions.

Second, we consider the continuity of the function f with respect to x and the continuity of the operator h in a generalized sense. This is so-called *D-continuity*, where D is a certain set in X^2.

Third, many of propositions are established in the situation without a measure. As in [3], [4], here the main role is played by a special system $\Re$ of subsets of the set T, by means of this system we introduce an analog of the notion "almost everywhere" in presence of a complete measure.

The article consists of three Sections. In Section 1 we expose starting premises and initial information on supmeasurability and standardness. Section 2 is devoted mainly to the Caratheodory D-conditions. It is shown that these are sufficient for both standardness and supmeasurability of the function f and D-continuity of the operator h in the sense of $\Re$-convergence, almost everywhere convergence, and convergence with respect to measure.

In Section 3 the main propositions of the article can be found, in which from the D-continuity of the operator h (in this or that sense) there are deduced the Caratheodory D-conditions for the generating h standard function f. In this situation, an essential role is played by V.L. Levin's theorem (see [9], [10]) on measurability of projection and measurable choice.

We use the following notation and abbreviations.

$:=$ is defining equality.

a. e. is "almost everywhere"; f. a. e. means "for almost each".

MsS, TS, MtS, and SMtS are, respectively, a measurable, topological, metric, and separable metric spaces.

$\operatorname{diag} X^2 := \{(x, u) \in X^2 \mid u = x\}$ is the diagonal in X^2.

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