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A Problem with Nonlocal Initial Data for One-Dimensional Hyperbolic Equation

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Abstract—In this paper we consider a boundary-value problem for one-dimensional hyperbolic equation with nonlocal initial data in integral form. We prove the existence and uniqueness of the generalized solution.

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Introduction. At the present time, problems with nonlocal conditions for partial differential equations are actively investigated. This interest is initiated by the necessity to generalize classical problems of Mathematical Physics in connection with the mathematical modelling of a series of physical processes, which are investigated by the modern natural sciences [1]. In those cases when the boundary of domain of physical process is not accessible for the immediate measuring, nonlocal conditions in the integral form can serve as additional information, which is sufficient for the unique resolvability of the corresponding mathematical problem. The present paper is dedicated to the investigation of solvability of a nonlocal problem with integral conditions.

An essential part of publications, starting from the G. R. Cannon paper [2], which are dedicated to problems with integral conditions, contains investigations of parabolic equations. Problems with integral conditions for hyperbolic equations are considered in [3–6] and in references therein. We note that in the majority of publications, which are dedicated to problems with nonlocal integral conditions, one considers spatially nonlocal conditions. In the present paper we consider the problem with nonlocal with respect to time integral conditions for a hyperbolic equation. Nonlocal problems with conditions of such a type for other equations were considered in [7–9]. The investigation of problems with nonlocal with respect to time integral conditions, whose results are presented in this paper, show that the dimension of a domain, in which one finds a solution, is important, and that the solvability conditions can connect both the domain dimensions and the constraints on other initial data.

Problems with nonlocal with respect to time conditions are tightly connected with inverse problems, whose condition of redefinition is integral one [10–12]. Thus defined conditions can be considered as a model of action of some apparatus, which registers physical fields [13]. As an example we adduce the condition of redefinition in the problem of definition of coefficient of parabolic equation [12]

$$\int_0^T \omega(\tau)u(x,\tau)d\tau = \chi(x).$$

One of investigation stages of resolvability of the inverse problem is often the solution to the direct problem, and in addition it is not excluded that the direct problem can be nonlocal. This reasoning has forced the purely theoretical interest to the investigation of nonlocal problems with integral conditions and has led to the definition of problem with nonlocal initial conditions for the one-dimensional hyperbolic equation.

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