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THE SHEA ESTIMATE FOR THE VALUE OF DEVIATION OF A MEROMORPHIC FUNCTION

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We investigate the influence of the number of divided points of the maximum of the module of a meromorphic function on a circle $\{z : |z| = r\}$ on the D. Shea estimate of the value of deviation. We obtain exact estimates of the corresponding values.

We shall use the standard notation applied in the Nevanlinna theory of distributions of values: $T(r, f)$, $m(r, a, f)$, $N(r, a, f)$ (see [1]).

After V.P. Petrenko's works, whose results were exposed in [2], the theory of the growth of meromorphic function started to develop. In this theory, for each $a \in \overline{\mathbb{C}}$, the approximation function for the number a is defined in the uniform metric

$$\mathcal{L}(r, a, f) = \max_{|z|=r} \log^+ \frac{1}{|f(z) - a|}, \quad \mathcal{L}(r, \infty, f) = \max_{|z|=r} \log^+ |f(z)|.$$

By means of this approximation function the following quantity is introduced

$$\beta(a, f) = \liminf_{r\to\infty} \frac{\mathcal{L}(r, a, f)}{T(r, f)},$$

which is called the (magnitude of) deviation of the function f with respect to the number a. Note that the Nevanlinna approximation function $m(r, a, f)$ measures the rate of approximation of f to a in the metric of $L_1[0, 2\pi]$, while the function $\mathcal{L}(r, a, f)$ does the same in the uniform metric. Therefore the Nevanlinna deficiency

$$\delta(a, f) = \liminf_{r\to\infty} \frac{m(r, a, f)}{T(r, f)}$$

does not exceed the value of deviation.

In 1976, A.F. Grishin in [3] constructed an example of a meromorphic function of an arbitrary order ρ, for which $\delta(\infty, f) = 0$, but $\beta(\infty, f) > 0$. Though the value $\beta(a, f)$ characterizes the approximation of f to the value a in the uniform metric, i. e., in a metric which is stronger than the metric of L_1, nevertheless, it turned out that, for meromorphic functions $f(z) \in \Phi(\lambda)$ possessing a finite lower order

$$\lambda := \liminf_{r\to\infty} \frac{\log T(r, f)}{\log r},$$

the properties of the values of deviation resemble the properties of the deficiencies. The first results of that type were obtained by V.P. Petrenko, who proved, in particular, the following