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THE ADMISSIBILITY OF PERIODIC PROCESSES AND EXISTENCE THEOREMS FOR PERIODIC SOLUTIONS. II

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Here we continue our research started in [1].

3. Periodic solutions of a nonlinear system

The Theorem which we shall state below differs from usual existence theorems by its profound geometric sense, thus allowing us to take into greater account the specific character of the system. On the other hand, the efficiency of Theorem in applications depends on how precisely we present a domain in the phase space of the system in which the supposed periodic solution lies.

We write the system in the form

$$\dot{x} = A(t,x)x + B(t,x)f(t,x), \tag{3.1}$$

where $A : \mathbb{R}^{1+n} \to \mathrm{Hom}(\mathbb{R}^n, \mathbb{R}^n)$, $B : \mathbb{R}^{1+n} \to \mathrm{Hom}(\mathbb{R}^n, \mathbb{R}^m)$, $f : \mathbb{R}^{1+n} \to \mathbb{R}^m$ are functions. Further, let $X : \mathbb{R} \to \Omega(\mathbb{R}^n)$ be a given function. In what follows we assume that

a) $t \to X(t)$ is a continuous function which has a period $T > 0$, and for any fixed t the set $X(t)$ is compact and convex in $\mathbb{R}^n$ (X is not supposed to be invariant with respect to the system (3.1));
b) for each fixed $x \in \mathbb{R}^n$, the functions A, B, and f are measurable with respect to t, bounded on $\mathbb{R}$, and T-periodic;
c) for each fixed t, the functions A, B, and f are locally Lipschitz at any point x from a certain neighborhood of the set $X(t)$.

Theorem 3. *If for arbitrary continuous T-periodic function $t \to \hat{x}(t) \in X(t)$ the system*

$$\dot{x} = A(t, \hat{x}(t))x + B(t, \hat{x}(t))u \tag{3.2}$$

is (U, X)-admissible, where $U(t) = f(t, X(t))$, then there exists at least one T-periodic solution $x(t)$ of system (3.1)*, where $x(t) \in X(t)$.*

Remark 6. A similar statement can be found in [2]: If for arbitrary continuous T-periodic function $t \to \hat{x}(t) \in X(t)$ the system $\dot{x} = A(t, \hat{x}(t))x$ is regular and for any point $(t_0, x_0) \in [0,T] \times \partial X(t_0)$ and for all $\xi \in N(t_0, x_0)$ there holds

$$\int_{t_0}^{t_0+T} \max_{x \in X(t)} \hat{\psi}(t,\xi)B(t,\hat{x}(t))f(t,x)dt \leqslant \xi x_0, \tag{3.3}$$

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