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ALMOST CONTACT MANIFOLDS ENDOWED WITH B-METRICS AND INTEGRABILITY

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We study the integrability conditions of the contact distributions for eleven classes of almost contact manifolds endowed with B-metrics.

1. Introduction

A manifold M of odd dimension $2n+1$ is called almost contact if the structure group of its tangent bundle can be reduced to the group $I \times U(n)$, where $U(n)$ is the unitary group, which is equivalent to the fact that M is endowed with a (φ, ξ, η)-structure (see [3]):

$$\eta(\xi) = 1, \quad \varphi^2 = -I + \eta \otimes \xi, \quad \varphi(\xi) = 0, \quad \eta \circ \varphi = 0.$$

The 1-form η determines a hyperplane in each tangent space of M. Thus M carries a $2n$-dimensional distribution, called the contact distribution of M.

In the case when on an almost contact manifold M a (Riemannian or pseudo-Riemannian) metric g is given which satisfies the relations

$$g(\varphi x, \varphi y) = -g(x, y) + \eta(x)\eta(y), \quad \text{(for arbitrary } x,\ y\text{)},$$

this manifold is called a special contact metric manifold. We denote such a manifold by $(M, g, \varphi, \xi, \eta)$ or, if there is no risk of ambiguity, by M. We denote by G_M the subgroup of the real representation of the group $I \times U(n)$ which preserves the metric g.

Contact metric manifolds of the above indicated type were considered by A. Kriščiūnaitė in [2] much earlier than in [1]. In [2], she studied hypersurfaces in a biaffine space, demonstrated that on a hypersurface of a biaffine space there arises a (φ, ξ, η)-structure, and proved that this (φ, ξ, η)-structure is integrable if and only if the hypersurface is a bicylinder. In [1], the term "time-like sphere" was used instead of "bicylinder".

The condition of compatibility of a metric g with a (φ, ξ, η)-structure can be rewritten in the form

$$g(\varphi x, y) = g(x, \varphi y), \tag{1}$$

which means that g is pure with respect to φ.

In [1], the following tensor field was introduced:

$$F(x, y, z) = g((\nabla_x \varphi)y, z),$$

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