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Stabilization of Periodic Systems with Aftereffect by Finite-Dimensional Approximations

Yu. F. Dolgii[*] and **E. V. Koshkin**[**]

Ural Federal University, ul. Mira 19, Ekaterinburg, 620002 Russia

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Abstract—We study the stabilization problem for a linear periodic system of differential equations with aftereffect. Approximating systems are described by differential equations with finite-dimensional Volterra operators. We construct admissible controls in the class of piecewise continuous functions by the feedback principle. We establish a connection between the approximating stabilization problem and that of the optimal stabilization for an autonomous linear system of difference equations.

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1. INTRODUCTION

The search of an optimal stabilizing control for a general class of systems of differential equations with aftereffect and a general set of admissible controls is rather difficult [1]. No analytical solution methods are known for this problem. Fundamental results of the approximation theory of the optimal stabilization are obtained in [2–5] for autonomous systems, and in [6–8] for periodic ones. In [9] one restricts the class of admissible controls to that of piecewise constant controls formed at discrete time moments by the feedback principle. Moreover, in the mentioned paper one proposes a method for constructing the optimal stabilizing control for systems with aftereffect defined by finite-dimensional operators. Such systems generalize systems of differential equations with piecewise constant arguments [10–13]. In this paper we develop the approximation theory of the stabilization of linear periodic systems of differential equations with aftereffect in a general form. As approximating systems we use systems with aftereffect defined by finite-dimensional operators. As the class of admissible controls for approximating systems we consider piecewise constant controls formed at discrete time moments by the feedback principle.

2. FINITE-DIMENSIONAL APPROXIMATIONS OF PERIODIC DIFFERENTIAL EQUATIONS WITH AFTEREFFECT

Consider the linear periodic functional differential equation

$$\frac{dx(t)}{dt} = (Fx)(t), \ \ t \in \mathbb{R}^+ = (0, +\infty),$$

stated in [14] (P. 8). Here $x : [-\tau, +\infty) \to \mathbb{R}^m$, the linear Volterra in the sense of the Tikhonov definition [15] operator $F : C([-\tau, +\infty), \mathbb{R}^m) \to L_1^{\text{loc}}((0, +\infty), \mathbb{R}^m)$ is such that $(Fx(\omega + \cdot))(t) = (Fx(\cdot))(\omega + t)$,

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[*] E-mail: `Jury.Dolgy@urfu.ru`.

[**] E-mail: `Koshkin@uralmail.com`.