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ON EQUATIONAL THEORIES OF CLASSES OF NONASSOCIATIVE RINGS

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The first example of a variety of universal algebras with unsolvable equational theory was given by A. Tarski (see [1]–[3]). A.I. Mal'tsev in [4] had built a finitely basable variety of quasigroups with unsolvable equational theory and put (see [5]) a question of existence of finitely basable varieties of semigroups, groups and rings with unsolvable equational theory. The affirmative answer to this question for groups was given in [6]. The first example of finitely basable variety of semigroups with unsolvable equational theory is given in [7]. Other examples of varieties of semigroups with unsolvable equational theory are described in [8] (theorem 3.16), [9], and [10]. In [11], it is announced that an arbitrary finitely basable variety of associative rings has a solvable equational theory. Examples of varieties of nonassociative rings with unsolvable equational theory are constructed in [12]–[17]. In [18], it is proved that a finitely basable variety of semigroups exists such that the equational theory of the class of all its finite semigroups is unsolvable. The varieties of semigroups satisfying this condition were described in [8] (theorem 3.17). In [19] the example of a finitely basable variety of nonassociative rings such that the equational theory of all its finite rings is unsolvable was constructed. In connection with these results, the following theorem is of certain interest.

Theorem. *A finitely basable variety of nonassociative rings with unsolvable equational theory exists such that the equational theory of the class of all its finite rings is unsolvable, too.*

The proof of this theorem is based on the interpretation of functioning of the bi-tape Minsky's machine. Note that for the first time this method was applied in [20], when proving the unsolvability of a quasiequational theory of classes of finite semigroups and associative rings. A detailed description of the methods, involving the interpretation of the Minsky's machines and a review of related results, is given in [8].

Let P be a recursively enumerable nonrecursive set of positive integers, and $\mathfrak{p}$ be a partial characteristic function of the set P [21]. Let us denote by M the bi-tape Minsky's machine computing the function $\mathfrak{p}$ [22]. Let us assume that q_0, q_1, $q_2, \ldots, q_m$ are the internal states of the machine M, where q_1 is the initial state and q_0 is the final state. If the machine is in the state q_i and the j-th tape is shifted by ξ_j cells to the left, we say that the configuration of M is $q_i\xi_1\xi_2$.

Let us introduce a number of denotations for words of a free groupoid Γ with a free generating set e, e_1, $e_2, \ldots, e_n, \ldots$. Set

$$s(x_1, \ldots, x_k) = ((x_{k-1}((\ldots(x_4((x_1x_2)x_3))x_5\ldots)x_{k-2}))x_k),$$
$$t(x_1, \ldots, x_{k+l}) = s(s(x_1, \ldots, x_k), x_{k+1}, \ldots, x_{k+l}).$$

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