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Pseudoconformally-Flat and Pseudo-Flat Quasi-Sasakian Manifolds

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Abstract—In this paper we consider the theory of pseudoconformally-flat (i.e., simultaneously contactly selfdual and contactly anti-selfdual) and pseudo-flat (i.e., simultaneously contactly R-selfdual and contactly R-anti-selfdual) 5-dimensional quasi-Sasakian manifolds.

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The role of conformally semiflat, i.e., selfdual or anti-selfdual, 4-dimensional manifolds (for example, [1–7]) in modern science is stipulated by direct connections of their geometric properties with those of Einstein manifolds [8] and with the twistor geometry [9]. The structural group of the principle bundle of oriented orthonormal frames over a 4-dimensional oriented Riemannian manifold M acts on the bundle of axisymmetric 2-forms over the manifold M, expanding the $C^\infty(M)$-module of $\Lambda_2(M)$ differential 2-forms on this manifold into the direct sum of two 3-dimensional submodules: $\Lambda_2(M) = \Lambda^+(M) \oplus \Lambda^-(M)$, i.e., the submodule of selfdual 2-forms and that of anti-selfdual 2-forms, respectively. It is well-known [8] that this property enables one to develop (with help of the Weyl tensor C considered as the symmetric automorphism of the module $\Lambda_2(M)$) the theory of conformally semiflat manifolds (it is called the *selfdual geometry* [6, 7]). In [10] the mentioned construction is extended onto 5-dimensional Riemannian manifolds endowed with an almost contact metric structure and therefore with a 4-dimensional hyperdistribution (i.e., a contact analog of conformally semiflat manifolds is introduced).

In this paper we consider the theory of pseudo-conformally flat (i.e., concurrently contactly selfdual and contactly anti-selfdual) and pseudoflat (i.e., concurrently contactly R-selfdual and contactly R-anti-selfdual) 5-dimensional quasi-Sasakian manifolds. The obtained results reveal interesting connections between pseudo-conformally flat and conformally flat manifolds of the mentioned type, as well as connections between pseudoflat and flat ones. In addition, we prove that 5-dimensional pseudoflat Sasakian manifolds do not exist.

1. Notions of pseudo-conformally flat and pseudoflat 5-dimensional almost contact metric manifolds. Let M be a smooth manifold; we denote by $C^\infty(M)$ the algebra of smooth functions on M, we do by $\mathfrak{X}(M)$ the $C^\infty(M)$-module of smooth vector fields on M. Recall ([11], P. 33) that the *almost contact metric* (briefly, the *AC*-) *structure* on M is the quadruple $(\Phi, \xi, \eta, g = \langle\cdot,\cdot\rangle)$ of tensor fields on M, where Φ is a tensor field of type $(1,1)$ that represents the endomorphism of the $C^\infty(M)$-module $\mathfrak{X}(M)$ and is called the *structural endomorphism*; ξ and η are vector and covector fields called the *characteristic vector* and the *contact form*, respectively; g is a bilinear symmetric positive definite form on $\mathfrak{X}(M)$ called the *Riemannian structure*. The following correlations take place: 1) $\eta(\xi) = 1$; 2) $\Phi(\xi) = 0$; 3) $\eta \circ \Phi = 0$; 4) $\Phi^2 = -\mathrm{id} + \eta \otimes \xi$; 5) $\langle \Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta(X)\eta(Y)$ for any $X, Y \in \mathfrak{X}(M)$. A manifold, on which the AC-structure is fixed, is said to be *almost contact metric* (briefly, an *AC*-) *manifold*.

Let $(M, \eta, \xi, \Phi, g = \langle\cdot,\cdot\rangle)$ be a 5-dimensional oriented AC-manifold. In the $C^\infty(M)$-module $\mathfrak{X}(M)$ two mutually complementary projectors are intrinsically defined: $\mathfrak{m} = \mathrm{id} + \Phi^2 = \eta \otimes \xi$ and $\mathfrak{l} = \mathrm{id} - \mathfrak{m} =$

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