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LINEAR ALGEBRAIC-DIFFERENTIAL SYSTEMS WITH VARYING DELAY

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1. Introduction

At present time, the systems of ordinary differential equations (ODE) with delay find very wide application, which can be equally related to algebraic-differential systems (ADS). Therefore it is completely naturally that one can note a permanently growing interest to the investigations in this domain. On the background of the increase of the quantity of works in every separate topic of this domain, we almost cannot cite investigations of the ADS with delay (some of the applications of such systems were given in [1]). This is partially related to the fact that even up to now not in all of the questions concerning ADS the limit clarity has been actually achieved (this is especially valid for nonlinear ADS of high index). Another reason for the present situation is that, in studying the ADS with delay, additional difficulties arise requiring the construction of a special theory.

Among the works concerning ADS with delay, we should note [2], [3], which were dedicated to the systems with constant coefficients

$$Ax'(t) = Bx(t) + Cx(t-\alpha) + Dx'(t-\alpha) + f(t), \quad t \geq 0, \quad \alpha = \text{const} > 0, \quad \det A = 0,$$
$$x(t) = \psi(t), \quad t \in [-\alpha, 0].$$

In the first of these works, in the conditions of the regularity of the bundle $\lambda A - B$ ($D = O$), the question of the existence of coordinated initial data was studied. In the second work, under the assumption that the bundle $\lambda A - B - \omega C - \lambda\omega D$ is regular ($\lambda A - B$ is singular), for this system a structure form was constructed which made it possible to obtain results about the behavior of the solution.

In this article, on the basis of existing theory of linear ADS (see, in particular, [4], [5]) we show that under some constraints upon the input data for the ADS with delay

$$A(t)y'(t) = B(t)y(t) + \sum_{i=0}^{p} D_i(t)y^{(i)}(t-\sigma(t)) + f(t), \quad t \in T = [t_0, t_k), \tag{1}$$
$$y(t) = \psi(t), \quad t \in [t_n, t_0), \tag{2}$$

$t_n = \min\limits_{t\in T}(t-\sigma(t))$, a linear differential operator exists transforming equation (1) to the form

$$y'(t) = P(t)y(t) + \sum_{j=1}^{s} G_j(t)y(t-\widetilde{\sigma}_j(t)) + \widetilde{f}(t), \quad t \in T,$$

for which the initial set remains same, i. e., $\min\limits_{t\in T,\ 1\leq j\leq s}(t-\widetilde{\sigma}_j(t)) = t_n$, and $y(t)$ for $t < t_0$ is determined by condition (2). Here $y(t)$ is an n-dimensional desired vector function, $A(t)$, $B(t)$, $D_i(t)$, and $G_j(t)$