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Existence and Basisness of Eigen and Adjoined Elements of Linear Operators

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Abstract—In this paper we continue the study described earlier in No. 5, 2006, of Russian Mathematics (Izv. Vyssh. Uchebn. Zaved., Matematika). We establish conditions, providing the asymptotics mentioned in the cited paper. We prove the basis property of eigen functions and adjoined ones in linear problems for differential equations with deviating arguments.

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1. INTRODUCTION

In this paper we continue the study described earlier in [1], where we prove that with a certain asymptotics (with $k \to +\infty$) of eigenelements $u_{(k,1)}$ of a linear operator $A : D_A \to H$ and those $u_{(k,2)}, \dots, u_{(k,n_k)}$ adjoined to $u_{(k,1)}$ one can choose the system

$$\{u_{(k,j)},\ j = 1, \dots, n_k,\ k = 1, 2, \dots\} \tag{1}$$

so that it forms a Riesz basis in the separable Hilbert space H. Elements $u_{(k,j)}$, $j = 1, \dots, n_k$, correspond to the eigenvalue λ_k and satisfy the correlations $\|u_{(k,1)}\| \neq 0$, $Au_{(k,1)} = \lambda_k u_{(k,1)}$, $Au_{(k,2)} = \lambda_k u_{(k,2)} + a_{k,1}u_{(k,1)}, \dots$, where $a_{k,j} \neq 0$ are certain (properly chosen) numbers. Hereinafter the abbreviation EAE means eigen and adjoined elements. Prior to [1] the basisness of (1) for abstract operators was proved only in the cases, when certain transforms of the operator A allowed one to apply the following theorems.

The Keldysh theorem ([2], P. 19):
if $\widehat{A}_0$ is a normal, complete, compact operator of a finite order, whose spectrum lies on a finite number of rays, and if $\widehat{A}_1 = T\widehat{A}_0$, T is compact, and $(I + T)$ is invertible, then the system of EAE of the operator $\widehat{A} = \widehat{A}_0 + \widehat{A}_1$ is complete in H.

The Markus theorem ([2], P. 28):
if the assumptions of the Keldysh theorem are fulfilled and if $\widehat{A}_1 = T_0(\widehat{A}_0)^{1+p}$ with certain $p \in (0, 1]$, T_0 is bounded, $(I + T_0(\widehat{A}_0)^p)$ is invertible, and

$$\overline{\lim_{r\to+\infty}}(r^{-p}n(r, \widehat{A}_0)) < +\infty,$$

then a certain system of EAE of the operator $\widehat{A}$ represents an unconditional basis with parentheses in H.

In addition, the system $\{v_{(r)},\ r = 1, 2, \dots\}$ is called a Riesz basis with parentheses in H, if one can find integer numbers $m_k < m_{k+1}$, $m_1 = 0$ and a Riesz basis in H $\{u_{(r)}, r = 1, 2, \dots\}$, such that $\{v_{(r)},\ r = 1, 2, \dots\} = \{\widehat{v}_{(q)},\ q = m_k + 1, \dots, m_{k+1},\ k = 1, 2, \dots\}$, $\{u_{(r)},\ r = 1, 2, \dots\} = \{\widehat{u}_{(q)},\ q =$

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