ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 9, pp. 1–6. © Allerton Press, Inc., 2010.
Original Russian Text © V.A. Abilov, F.V. Abilova, M.V. Abilov, 2010, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2010, No. 9, pp. 3–9.

Some Inverse Theorems for Approximation of Functions by the Fourier–Laguerre Sums

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Received December 11, 2008

Abstract—In this paper we prove two inverse theorems for approximation of functions of two variables by Fourier–Laguerre sums in the space $L_2(\mathbb{R}_+^2; x^\alpha y^\beta e^{-x-y})$.

DOI: 10.3103/S1066369X1009001X

Key words and phrases: *approximation by Fourier–Laguerre sums, generalized modulus of continuity, functions of two variables.*

Let $L_2 = L_2(\mathbb{R}_+^2; x^\alpha y^\beta e^{-x-y})$ $\left(\alpha, \beta > -\frac{1}{2}\right)$ be the space of square summable functions f of two variables in the domain $\mathbb{R}_+^2 = (0, +\infty) \times (0, +\infty)$ with the weight $x^\alpha y^\beta e^{-x-y}$ and the norm

$$\|f\| = \sqrt{\int_0^{+\infty} \int_0^{+\infty} x^\alpha y^\beta e^{-x-y} f^2(x, y) dx\, dy}.$$

For a function $f \in L_2$ we denote by $E_N(f) = \inf_{P_N} \|f - P_N\|$ its best approximation by algebraic polynomials

$$P_N(x, y) = \sum_{0 \leqslant n+m < N} a_{nm} x^n y^m \quad (N = 1, 2, \dots)$$

in the space L_2.

In the space L_2 we consider the following operator:

$$F_h f(x, y) = \frac{2^{\alpha+\beta}\Gamma(\alpha+1)\Gamma(\beta+1)}{2\pi} \times \int_{-1}^{1}\int_{-1}^{1} f\big(x + h + 2u\sqrt{xh},\ y + h + 2v\sqrt{yh}\big) K_\alpha(h, x, u) K_\beta(h, y, v) du\, dv$$

$$\left(K_\nu(h, s, t) = \Im_{\nu-\frac{1}{2}}\big(\sqrt{hs(1-t^2)}\big)\big(\sqrt{hs(1-t^2)}\big)^{-\nu-\frac{1}{2}} e^{-t\sqrt{hs}}(1-t)^{\nu-\frac{1}{2}}\right),$$

where $\Im_p(u)$ is the Bessel function of the first kind of order p. We call it the *generalized shift operator* in the space L_2.

Let us define finite differences of the first and higher orders

$$\Delta_h(f; x, y) = F_h f(x, y) - f(x, y) = (F_h - E) f(x, y),$$

$$\Delta_h^k(f; x, y) = \Delta_h(\Delta_h^{k-1}(f; x, y); x, y) = (F_h - E)^k f(x, y) = \sum_{i=0}^{k} (-1)^{k-i} \binom{k}{i} F_h^i f(x, y),$$

where $F_h^0 f(x, y) = E f(x, y) = f(x, y)$, $F_h^i f(x, y) = F_h(F_h^{i-1} f(x, y))$ $(i = 1, 2, \dots, k;\ k = 1, 2, \dots)$, and E stands for the unit operator in the space L_2.