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ON MODIFICATION OF THE PROJECTION SCHEME OF SOLVING ILL-POSED PROBLEMS

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We consider a problem of discretization for some classes of equations of first genus $Ax = f$ with solutions $x = |A|^p v$ representable as sources. For any $p > 0$, we construct a method which, in the case of an a priori choice of regularization parameter, attains an optimal order of accuracy. Besides, we show that with respect to the volume of the used discrete information the suggested method is more economic in comparison with traditional discretization by Galyorkin.

In the present article we suggest an economic modification of projection scheme of solving ill-posed problems. This scheme allows to attain an optimal order of accuracy while the volume of discrete information is essentially less than that for traditional methods.

Let X be a Hilbert space, and A be a linear compact operator acting from X to X. We consider the problem of finite-dimensional approximation of solutions of equations of first genus

$$Ax = f \tag{1}$$

with the free term f from the set $\mathrm{Range}(A) := \{f : f = Ag,\ g \in X\}$. We shall assume that instead of f its certain approximation $f_\delta \in X_{\delta,f}$ is given, where $X_{\delta,f}$ is a ball of the radius δ in the space X with the center in f, while δ is a small positive number, which is usually unknown.

Approximate solution of (1) will be constructed by means of a regularizator for this equation. In following the idea from [1] on the construction of the regularizator as a function of the operator of the equation to be solved, we consider a certain parametric family of functions $\{g_\alpha\}$, $0 < \alpha < 1$, which are Borel measurable on the segment $[0, \gamma_1^2]$, $\|A\|_{X\to X} \le \gamma_1$. We impose two conditions upon these functions

$$\sup_{0\le\lambda\le\gamma_1^2} \lambda^p |1 - \lambda g_\alpha(\lambda)| \le \chi_p \alpha^p, \quad 0 \le p \le p_0, \tag{2}$$

$$\sup_{0\le\lambda\le\gamma_1^2} \lambda^{1/2} |g_\alpha(\lambda)| \le \chi_* \alpha^{-1/2}, \tag{3}$$

where p_0, χ_p, and χ_* are some positive constants independent of α. Let x_0 be the least by its norm in X solution of (1). Then in the capacity of an approximation to x_0 we take the element $x_\alpha = R_\alpha f_\delta$, where the regularizator R_α, $\alpha = \alpha(\delta)$, will be given by the relation

$$R_\alpha = R_\alpha(A) =: g_\alpha(A^*A)A^*, \tag{4}$$

and A^* is such that with any $\varphi, g \in X$ we have $(\varphi, Ag) = (A^*\varphi, g)$ (by $(\cdot,\cdot)$ we understand the scalar product in X).

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