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MINIMAL k-EXTENSIONS OF PRECOMPLETE GRAPHS

M.B. Abrosimov

1. Introduction

A nonoriented graph (in what follows: a graph) is a pair $G = (V, \alpha)$ in which V is a set, called the set of vertices, and α is a symmetric and antireflexive relation on V. *The degree of a vertex* v in a nonoriented graph is the number of vertices adjacent to v. We denote the degree of v by $d(v)$. The collection of numbers which are the degrees of vertices of a graph G is called *the degree set*, the vector consisting of the degrees of all vertices of a graph in descending order is called *the degree vector*. A graph is said to be *a realization* of its degree vector. A graph is called *a unigraph* if it is the unique realization of its degree vector. A graph all of whose vertices have the same degree p is called a homogeneous or regular graph of order p. Here and in what follows the definitions are taken from [1].

A subgraph of a graph $G = (V, \alpha)$ is a pair $G' = (V', \alpha')$ such that $V' \subseteq V$ and $\alpha' = (V' \times V') \cap \alpha$. *An embedding of a graph* $G_1 = (V_1, \alpha_1)$ into a graph $G_2 = (V_2, \alpha_2)$ is an injective mapping $f : V_1 \to V_2$ such that, for any $u, v \in V_1$, the following condition holds: $((u, v) \in \alpha_1) \Rightarrow ((f(u), f(v)) \in \alpha_2)$.

A graph $G_3 = (V_3, \alpha_3)$ is called a *k-extension of a graph* $G = (V, \alpha)$ if G can be embedded into each subgraph of G_3 obtained by removing any k of its vertices together with the incident edges.

The trivial k-extension of a graph $G = (V, a)$ is the graph $G_t = (V_t, a_t)$ obtained by adding k vertices to G and joining them with all vertices of G and with each other, i. e., G_t is the join of G and the complete graph $K_k = (V_k, V_k \times V_k)$. Obviously, the trivial k-extension of a graph is also its k-extension and $|V_t| = |V| + k$.

A graph $G^* = (V^*, \alpha^*)$ is called *a minimal k-extension* of a graph $G = (V, \alpha)$ if the following conditions are fulfilled:

1) G^* is a k-extension of G;

2) $|V^*| = |V| + k$;

3) α^* has the minimal cardinality provided that conditions 1) and 2) are fulfilled.

The following statements are immediate.

Lemma 1. *Any minimal k-extension of a connected graph contains no vertices with degree less than $(k + 1)$.*

Lemma 2. *Let s be the greatest degree of the vertices of a graph G and exactly m vertices have degree s. Then a minimal k-extension of G contains at least $(k + m)$ vertices with degree not less than s.*

Lemma 3. *If the maximum degree of the vertices of a graph G is d, then its minimal k-extension contains not less than kd additional edges.*