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EQUATIONS DESCRIBING DYNAMICS OF NON-NEWTONIAN FLUID WITH REINER–RIVLIN RHEOLOGICAL LAW. I. GROUP ANALYSIS

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1. Introduction

In the first part of the present paper we give complete group analysis of the equation system

$$u_t = (u^2 f(v))_x, \quad v = uu_x. \tag{1.1}$$

Equations (1.1) describe dynamics of surface of non-Newtonian fluid subordinate to the Reiner–Rivlin rheological law [1], the process of turbulent gas filtration in porous medium [2], the processes of fluid shocks, and etc. In applications, the arbitrary function f always satisfies

$$\textit{Condition } A: \quad f(0) = 0$$

(according to the terminology in [3], which we will follow, this function is called an arbitrary element).

Presently, a number of equations with arbitrary element, which are of considerable interest for applications, are completely studied from the group analysis point of view. Among them there are the gas dynamics equation system [3]–[6], the Prandtl equation of boundary layer, the equation of nonlinear head conduction without source and with source depending on temperature, the equation of nonlinear filtration [3], [7]. The problem of group analysis of nonlinear wave equation was also partially solved [8] (we refer the reader to [1] for sufficiently complete review of obtained results). The aim of the present paper is to add (1.1) to this list. In the second part of the present paper we study invariant solutions of the invariant equations obtained in the first part. If Condition A holds true, equation (1.1) is an equation with double degeneracy (with respect to the unknown function and its derivative). Note that properties of solutions of these equations are now actively studied [9].

It is known [3] that for the equations with arbitrary element the complete group analysis should contain 1) description of the basic algebra of equation (1.1) with arbitrary f; 2) description of the group of its equivalence transformations; 3) classification of all possible extensions of the basic algebra (up to transformations in the equivalence group) and descriptions of these extensions. The general method for solving these problems was developed by S. Lie, however while applying this method to each concrete problem one meets substantial difficulties. This explains why the list of completely solved problems given above is not so large.

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