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A Nonlocal Problem for Degenerate Hyperbolic Equation

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Abstract—We consider a nonlocal problem for a degenerate equation in a domain bounded by characteristics of this equation. The boundary-value conditions of the problem include linear combination of operators of fractional integro-differentiation in the Riemann–Liouville sense. The uniqueness of solution of the problem under consideration is proved by means of the modified Tricomi method, and existence is reduced to solvability of either singular integral equation with the Cauchy kernel or Fredholm integral equation of second kind.

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1. Statement of the problem. We consider the equation

$$|y|^m u_{yy} - u_{xx} + \alpha \operatorname{sign} y |y|^{m-1} u_y = 0, \tag{1}$$

where $\alpha = \text{const}$, $0 < m < 1$, in a finite domain Ω bounded by characteristics

$$AC: x - \frac{2}{m-2} y^{\frac{2-m}{2}} = 0, \quad BC: x + \frac{2}{m-2} y^{\frac{2-m}{2}} = 1,$$

$$AD: x - \frac{2}{m-2} (-y)^{\frac{2-m}{2}} = 0, \quad BD: x + \frac{2}{m-2} (-y)^{\frac{2-m}{2}} = 1$$

of Eq. (1). Let $\Omega_1 = \Omega \cap (y > 0)$, $\Omega_2 = \Omega \cap (y < 0)$, I be an interval $0 < x < 1$ of axis $y = 0$.

Problem. Find the solution

$$u(x,y) = \begin{cases} u_1(x,y), & (x,y) \in \Omega_1; \\ u_2(x,y), & (x,y) \in \Omega_2, \end{cases}$$

to Eq. (1) in the class $C(\overline{\Omega}) \cap C^1(\Omega_1 \cup I) \cap C^1(\Omega_2 \cup I) \cap C^2(\Omega_1 \cup \Omega_2)$ satisfying conditions

$$a_i(x) D_{0x}^{\beta_i} \delta_i(x) u_i[\Theta_0^i(x)] + b_i(x) D_{x1}^{\beta_i} w_i(x) u_i[\Theta_1^i(x)] + c_i(x) u_i(x,0) + d_i(x) u_{i\,y}(x,0) = f_i(x), \quad i = 1, 2, \tag{2}$$

and conjugation condition

$$\lim_{y\to+0} y^\alpha u_y(x,y) = \lim_{y\to-0} (-y)^\alpha u_y(x,y), \tag{3}$$

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