

## ON APPROXIMATION OF SEMIGROUPS OF ISOMETRIES IN A HILBERT SPACE

G.G. Amosov

We investigate a class of  $C_0$ -semigroups of isometric operators  $(U_t)_{t \geq 0}$ , connected with a fixed  $C_0$ -semigroup of isometric operators  $(V_t)_{t \geq 0}$  in a Hilbert space with the condition  $\|U'_t - V'_t\|_2 = O(t^{1/2})$ ,  $t \rightarrow 0$ . Here  $(U'_t)_{t \geq 0}$  and  $(V'_t)_{t \geq 0}$  are minimal unitary dilations of initial semigroups,  $\|\cdot\|_2$  is the Hilbert–Schmidt norm. The condition introduced above is sufficient for cocyclic conjugacy of semigroups of endomorphisms of the algebra of canonical anticommutative relations determined by the semigroups  $(U_t)_{t \geq 0}$  and  $(V_t)_{t \geq 0}$ . In Section 1, a definition of an approximation of  $C_0$ -semigroups of isometric operators in a Hilbert space is given and its consequences are exposed. In Section 2 it is proved that, among all the semigroups of isometric operators approximating a given semigroup with a uniformly continuous unitary part, a semigroup of non-unitary completely isometric operators can be found.

### 1. Definition of approximation

In what follows we consider  $C_0$ -semigroups  $(V_t)_{t \geq 0}$  of isometric operators in a separable Hilbert space  $h$ . We denote by  $s_2$ ,  $\|\cdot\|_2$ ,  $s_\infty$ , and  $\|\cdot\|$  the classes of Hilbert–Schmidt operators, the Hilbert–Schmidt norm, the class of completely continuous operators in  $h$ , and the norm in  $B(h)$ , respectively. By a generator of a  $C_0$ -semigroup  $(V_t)_{t \geq 0}$  we call, in general, an unbounded skew-symmetric operator  $d = \lim_{t \rightarrow 0} \frac{V_t - I}{t}$  with the domain  $\mathcal{D}(d)$  dense in  $h$  (see [1], Chap. III, p. 160).

**Definition.** A  $C_0$ -semigroup  $(V_t)_{t \geq 0}$  of isometric operators in the Hilbert space  $h$  is said to be *approximating the  $C_0$ -semigroup  $(U_t)_{t \geq 0}$  of isometric operators in  $h$*  if for the semigroups  $(V_t)_{t \geq 0}$  and  $(U_t)_{t \geq 0}$  the minimal unitary dilations  $(V'_t)_{t \geq 0}$  and  $(U'_t)_{t \geq 0}$ , acting in the Hilbert space  $h'$ ,  $h \subset h'$ , can be found such that

$$\|V'_t - U'_t\|_2 = O(t^{1/2}), \quad t \rightarrow 0, \quad (1)$$

where  $V'_t - U'_t = \Delta_t \in s_2$ ,  $t \geq 0$ , is a family of operators continuous with respect to the norm  $\|\cdot\|_2$ .

**Remark 1.** Condition (1) implies that the semigroups approximating each other in the sense of our definition will approximate each other in the sense of the definition in [2], i.e., they obey the relation  $\|U_t - V_t\| = O(t^{1/2})$ ,  $t \rightarrow 0$ .

**Remark 2.** Any  $C_0$ -semigroup of isometric operators  $(V_t)_{t \geq 0}$  in a Hilbert space  $h$  determines a semigroup of endomorphisms  $(B(V_t))_{t \geq 0}$  of the algebra of canonical anticommutative relations  $\mathcal{A}(h)$  over  $h$ . For the cocyclic conjugacy of the semigroups of endomorphisms  $(B(U_t))_{t \geq 0}$  and  $(B(V_t))_{t \geq 0}$  it suffices that  $(U_t)_{t \geq 0}$  approximate  $(V_t)_{t \geq 0}$  (see [3]).

---

©2000 by Allerton Press, Inc.

Authorization to photocopy individual items for internal or personal use, or the internal or personal use of specific clients, is granted by Allerton Press, Inc. for libraries and other users registered with the Copyright Clearance Center (CCC) Transactional Reporting Service, provided that the base fee of \$50.00 per copy is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923.