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A Nonstationary Group Pursuit Problem

A. S. Bannikov[1*]

[1] *Udmurt State University, 1 Universitetskaya str., Izhevsk, 426034 Russia*

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Abstract—We consider a linear nonstationary problem of conflict interaction of controlled objects, where the number of pursuers equals ν and the number of evaders equals μ. All participants are assumed to have equal dynamic abilities. The purpose of the pursuers is to catch all evaders, while the purpose of the latter is to avoid being caught for at least one of them. We establish sufficient solvability conditions for the local evasion problem.

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1. PROBLEM STATEMENT

In the space $\mathbb{R}^n$ $(n \geqslant 2)$ we consider a differential game Γ of $\nu + \mu$ players, namely, ν pursuers and μ evaders. The law of motion of each pursuer P_i, $i = 1, \ldots, \nu$, is

$$\dot{x}_i(t) = A(t)x_i(t) + u_i(t), \;\; x_i(t_0) = x_i^0, \;\; u_i \in U.$$

The law of motion of each evader E_j, $j = 1, \ldots, \mu$, takes the form

$$\dot{y}_j(t) = A(t)y_j(t) + v_j(t), \;\; y_j(t_0) = y_j^0, \;\; v_j \in U;$$

in addition, $x_i^0 \neq y_j^0$ for all i, j. Here x_i, y_j, u_i, $v_j \in \mathbb{R}^n$, $U \subset \mathbb{R}^n$ is a convex compact, $A(t)$ is a real square matrix of order n measurable on the whole axis t, the norm $\|A(t)\|$ is integrable on any compact subset of the axis t. Control actions of the players are measurable functions $u_i(t)$, $v_j(t)$; with $t \geqslant t_0$ they take on values in the set U.

Definition 1. We say that in the differential game Γ from the initial state $z^0 = (x_1^0, \ldots, x_\nu^0, y_1^0, \ldots, y_\mu^0)$ the *local evasion problem* is *solvable* on the semi-infinite interval $[t_0, +\infty)$, if there exist evaders' controls $v_1(t), \ldots, v_\mu(t)$ such that with any pursuers' controls $u_1(t), \ldots, u_\nu(t)$ one can find a number $s \in \{1, \ldots, \mu\}$ such that $y_s(t) \neq x_i(t)$ for all $i \in \{1, \ldots, \nu\}$ and all $t \geqslant t_0$.

2. GEOMETRIC PROPERTIES OF THE ATTAINABILITY SET

Consider a controlled object whose behavior obeys the equation

$$\dot{x}(t) = A(t)x(t) + u(t), \;\; u \in U. \tag{2.1}$$

Let $x(t)$ be a solution to Eq. (2.1) that corresponds to the control $u(t)$ and the initial condition $x(t_0) \in M_0$, where M_0 is a convex compact. Let $X(t; t_0, M_0, U)$ stand for the attainability set of the controlled object at the moment $t \geqslant t_0$ from the set M_0, let $C(V; \psi)$ denote the support function of the compact V.

Definition 2. The pair $\{u(t), x(t)\}$ is said to satisfy the maximum condition on the segment $[t_0, t_1]$ and the transversality condition on the set M_0, if there exists a solution $\psi(t)$ to the adjoint system $\dot{\psi}(t) = -A^*(t)\psi(t)$ with the initial condition $\psi(t_0) \in \partial S$ such that the following conditions are fulfilled:

*E-mail: bannikov_a_s@mail.ru.