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STABLE SOLVABILITY OF MONOTONE AND ACCRETIVE OPERATORS

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This article is devoted to the investigation of solvability of the operator equation

$$Tx = Fx \tag{1}$$

with a continuous operator $T : X \to Y$ and a completely continuous operator $F : X \to Y$, X and Y being Banach spaces.

Equation (1) was investigated by various authors in the cases where T is a linear operator (so-called quasilinear equations) or where T admits a linearization (for example, is a differentiable operator). However, a few works study equation (1) with an essentially nonlinear operator T, i. e., without assuming that it can be linearized.

In this article we suggest a method for investigation of the solvability of equation (1), which is based on a new concept of "stable solvability" with respect to a certain class of perturbations. Earlier, in [1], an attempt was made to introduce such a concept.

In this article the main attention will be paid to the study of conditions for the stable solvability of monotone by Minty–Brauder and accretive operators.

We denote by Φ a nonempty convex set of continuous operators $H : X \to Y$. Let $G \subset Y$ be a subset of the space Y.

Definition 1. A continuous operator $T : X \to Y$ is said to be stably solvable with respect to a class Φ and a set G (or (Φ, G)-stably solvable) if, for any operator $H \in \Phi$ and an arbitrary element $g \in G$, the equation

$$Tx = Hx + g$$

has at least one solution.

This definition possesses a certain generality because it contains an arbitrariness in the choice of the class Φ and the set G. Rich in content assertions concerning the stable solvability can be obtained with respect to the intrinsic class

$$\Phi\,(X,Y) = \{H : X \to Y \text{ is completely continuous and such that } \lim_{r\to\infty} \sup_{\|x\|=r} \|Hx\| = 0\}.$$

We shall also assume that the set G coincides with the whole space X. In this case, we shall simply speak about the stable solvability. The property of the stable solvability is preserved under completely continuous perturbations of a certain type. This information is supplied by the following

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