ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2011, Vol. 55, No. 5, pp. 56–62. © Allerton Press, Inc., 2011.
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Functional Conditions for the Convergence of Fourier Series with Respect to General Orthonormal Systems

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Received November 10, 2009

Abstract—In this paper we establish some functional conditions for the convergence of the Fourier series for functions from the class $C(0,1)$ with respect to general orthonormal systems.

DOI: 10.3103/S1066369X11050094

Keywords and phrases: *orthonormal system, Fourier series, Fourier coefficients, modulus of continuity.*

1. INTRODUCTION

A functional condition imposed on some function ensures the convergence of its Fourier series with respect to some general orthonormal systems (ONS).

It is well-known [1] that if $f \in L_2(0,1)$ $(f \not\equiv 0)$ is some function, then there exists an orthonormal on $[0,1]$ system, with respect to which the Fourier series of the function f diverges a. e. on $[0,1]$. Therefore, even for the function $f \equiv 1$ the convergence of the Fourier series with respect to ONS is not guaranteed. In such a case, if one needs the convergence of the Fourier series of functions from some functional class with respect to some ONS, this ONS must satisfy certain conditions.

2. AUXILIARY DENOTATIONS AND THEOREMS

Let (φ_n) be an ONS on $[0,1]$. The Fourier coefficients of functions $f \in L_1(0,1)$ are defined as follows:

$$\widehat{\varphi}_n(f) = \int_0^1 f(x)\varphi_n(x)\,dx, \quad n = 1, 2, \dots$$

The expression

$$\sum_{n=1}^{\infty} \widehat{\varphi}_n(f)\varphi_n(x) \tag{1}$$

is the Fourier–Lebesgue series of the function f. Let

$$S_n(f;x) = \sum_{k=1}^{n} \widehat{\varphi}_k(f)\varphi_k(x)$$

be a partial sum of this series. The Cesàro average of series (1) takes the form

$$\sigma_n^\alpha(f;x) = \frac{1}{A_n^\alpha}\sum_{k=0}^{n} A_{n-k}^{\alpha-1} S_k(f;x), \quad A_n^\alpha = \binom{n+\alpha}{n}.$$

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