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A SUPPLEMENT TO THE JAMET TEST

A.V. Antonova

This paper deals with new convergence tests for positive series which simplify the study of the latter.

Let three functions $\varphi(n)$, $g(n)$, $f(n)$ be defined on the set $\mathbf{N}$ of positive integers and satisfy the conditions

1) $\varphi(n)$, $f(n)$ are increasing, positive defined and nonbounded functions;
2) $g(n)$ is positive defined;
3) $\lim\limits_{n\to\infty} \frac{\varphi(n)g(n)}{f(n)\ln n} = q$;
4) $\lim\limits_{n\to\infty} \frac{g(n)}{f(n)} = 0$.

Let us consider the cases, where $q \neq 0$ is finite, $q = 0$, and $q = +\infty$, separately.

Assume that $q \neq 0$ is finite. Then the above assumptions ensure the validity of the following assertion.

Theorem 1. *If the terms of a series* $\sum\limits_{n=1}^{\infty} a_n$ *for certain* N *satisfy* $\forall n \geq N$ *the inequality*
1) $(1 - a_n^{1/\varphi(n)})f(n)/g(n) \geq p > 1/q$, *where* p *is a constant, then the series converges.*
Vice versa, if $(\forall n \geq N)$ *the following inequality is valid:*
2) $(1 - a_n^{1/\varphi(n)})f(n)/g(n) \leq p < 1/q$ *for a constant value* p *then the series diverges.*

Proof. Assume that inequality 1) of Theorem 1 is valid for any $n \geq N$, beginning from certain N. Then

$$(1 - pg(n)/f(n))^{\varphi(n)} \geq a_n. \tag{1}$$

Let us prove that the series

$$\sum_{n=k}^{\infty} (1 - pg(n)/f(n))^{\varphi(n)} \tag{2}$$

converges. Here k is a number such that all terms of the series are defined for it and larger numbers. Let us apply the logarithmic test [1] to this series

$$\frac{\ln(1 - pg(n)/f(n))^{-\varphi(n)}}{\ln n} = \frac{\varphi(n)pg(n)}{f(n)\ln n} \ln B(n), \quad B(n) = (1 - pg(n)/f(n))^{\frac{f(n)}{-pg(n)}}. \tag{3}$$

By assumption 4) we obtain $\lim\limits_{n\to\infty} \ln B(n) = 1$. Consequently, for any $\varepsilon > 0$ we can find N_1 such that for any $n \geq N_1$ the inequality

$$1 - \varepsilon < \ln B(n) < 1 + \varepsilon \tag{4}$$