*Russian Mathematics (Iz. VUZ)* *Izvestiya VUZ. Matematika*
*Vol. 48, No. 5, pp. 22–29, 2004* *UDC 514.764*

# WEYL GEODESIC FIELD OF CONES IN A THREE-DIMENSIONAL RIEMANNIAN SPACE

O.S. Germanov

The notion of a geodesic field of cones of directions in a Riemannian space was introduced in [1]. A field of cones is defined with the use of a symmetric tensor $a_{ij}(x^k)$ ($x^k$ are coordinates in the space; $i,j,k=\overline{1,n}$; $n$ is the dimension of the space) by the equation

$$a_{ij}X^iX^j=0, \tag{1}$$

where $X^i$ are the components of a contravariant vector determining a direction. A field of cones (1) is called geodesic if all geodesics whose directions coincide at some point with a direction satisfying (1) retain this property at all their points. As was shown in [1], the following equalities are a criterion for a field of cones (1) to be a geodesic one: $\nabla_{(k}a_{ij)}=M_{(k}a_{ij)}$, where $M_k$ are the components of some vector, $\nabla_k$ is the symbol of covariant differentiation with respect to the connection under consideration, and the brackets denote the symmetrization with respect to the corresponding indices.

In [2], fields of cones (1) in a Riemannian space were considered whose nondegenerate tensor together with some vectors $M_k$ and $R_k$ and the metric tensor $g_{ij}$ satisfy the equation

$$\nabla_{(k}a_{ij)}=M_{(k}a_{ij)}+R_{(k}g_{ij)}. \tag{2}$$

In [3], such fields were called Weyl geodesic. Riemannian spaces admitting Weyl geodesic fields of cones generated by a nondegenerate tensor field $a_{ij}$ of a special type whose characteristic equation

$$|a_{ij}-\lambda g_{ij}|=0 \tag{3}$$

has at least two real roots were constructed in [4].

In this paper, we construct three-dimensional Riemannian spaces admitting a special (see § 2 below) Weyl geodesic field of cones whose characteristic equation has a pair of complex conjugate roots or a real root of multiplicity 3 (the latter case is excluded in [4]).

## 1. Weyl geodesic field of cones

Consider some properties of Weyl geodesic fields of cones.

First of all, the following should be noted: although the tensor of such a field is defined up to an arbitrary functional factor, the geometrical properties of this field do not depend on the choice of a normalization of its tensor.

In fact, for a tensor $\widetilde{a}_{ij}$ such that $a_{ij}=\alpha\widetilde{a}_{ij}$ ($\alpha$ is a function of the coordinates), from (2) it follows that $\nabla_{(k}\widetilde{a}_{ij)}=\widetilde{M}_{(k}\widetilde{a}_{ij)}+\widetilde{R}_{(k}g_{ij)}$, where $\widetilde{M}_k=M_k-\alpha^{-1}\partial_k\alpha$, $\widetilde{R}_k=\alpha^{-1}R_k$, $\partial_k\equiv\frac{\partial}{\partial x^k}$. In addition, the form of equations (2) is invariant with respect to conformal transformations ([5], p. 161) of connection.