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ON THE DYNAMICS OF A MONOTONE MAPPING OF n-OD

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1. The papers [1]–[5] are related to the questions of coexistence of periods of periodic points, positiveness of the topological entropy of continuous mappings of finite graphs. In [6], there was studied the dynamics of primitive mappings of dendrites which admit not only a finite, but also countable set of branching points. There were cited examples illustrating the difference between entropy properties of the continuous mappings of dendrites with a countable set of branching points and the continuous mappings of their retracts — finite trees.

The present article continues [6]. We shall consider here the basic aspects of the dynamics of monotone mappings of an elementary representative of the dendrite class — an n-od.

2. Let us cite necessary definitions and formulate the main results of the article.

Definition 1. Let C be the complex plane, n be an arbitrary natural number. By an n-od we call the set $X = \{z \in C : z = |z|e^{i\frac{2\pi}{n}(j-1)},\ 0 \le |z| \le 1,\ j = 1, 2, \ldots, n,\ i$ is the imaginary unit$\}$.

X has the unique point of branching 0, and an open in X set $X\setminus\{0\}$ is composed of n components called the components of the n-od. Let us agree to concord the numeration of components with the angular coordinate of their points and denote by X_j a component of the n-od, whose arbitrary point's angular coordinate equals $\frac{2\pi}{n}(j-1)$, $1 \le j \le n$.

The closure $\overline{X_j}$ of an arbitrary component X_j, $1 \le j \le n$, is called a branch of the n-od.

By an arc in X we shall understand the set homeomorphic to a segment on the straight line. Assumed to be a degenerate arc, a one-point set thus is also related to the class of arcs. We shall denote by $\gamma(x, y)$ an arc with the ends at the points x and y and which contains these points. For an arbitrary $1 \le j \le n$, there exists a point $e_j \in X_j$ such that $\overline{X_j} = \gamma(0, e_j)$. The points e_j, $1 \le j \le n$ are called endpoints of the n-od.

Definition 2 ([6]). A mapping $f : X \to X$ is said to be primitive if f is continuous and the complete preimage of any arc from $f(X)$ is an arc in X.

We denote by $P^0(X)$ the set of all primitive mappings of an n-od into itself. For whatever $f \in P^0(X)$, for any $m \ge 1$ we have $f^m \in P^0(X)$. The example of a primitive mapping is supplied by a rotation of an n-od X by the angle $2\pi/n$ with the fixed point 0.

Let us note that though the n-od is not a linearly ordered topological space (with the topology induced by the topology of the complex plane), nevertheless the notion of a monotone mapping of the n-od into itself is correctly defined.

Definition 3 ([7], p. 140). A mapping $f : X \to X$ is said to be monotone if f is continuous and the complete preimage of any connected set from $f(X)$ is a connected set.

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