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Singularly Perturbed Dirichlet Boundary-Value Problem for a Stationary System in the Linear Elasticity Theory

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Abstract—We consider a singularly perturbed Dirichlet boundary-value problem for an elliptic operator of the linear elasticity theory in a bounded domain with a small cavity. The main result is the proof of the theorem about the convergence of eigenelements of the perturbed boundary-value problem to eigenelements of the corresponding limiting boundary-value problem, when the parameter ε which defines the diameter of the small cavity tends to zero.

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Many mathematicians studied the boundary-value problems for elliptic operators in a domain with a small cavity. As far back as 1948 in [1] A. A. Samarskii proved that a perturbation of an eigenvalue λ_n of the Dirichlet problem for the Laplace operator in a domain $\Omega \subset \mathbb{R}^3$ without a small set ω_ε admits the following bound:

$$\triangle\lambda_n \le \varphi_n^2 \alpha(\omega_\varepsilon;\Omega) + O(\alpha^2(\omega_\varepsilon;\Omega)),$$

where φ_n is the maximal value of the normalized nth eigenfunction on $\overline{\omega}_\varepsilon$, $\alpha(\omega_\varepsilon;\Omega)$ tends to zero as $\varepsilon \to 0$ and depends on geometric properties of the domains ω_ε and Ω. An analogous result was obtained later in [2]. In [3] A. M. Il'in described an asymptotic form of a solution to a scalar boundary-value problem for an elliptic operator of the second order in an n-dimensional domain without a small subset. In [4] he constructed an asymptotic form of a solution to a perturbed elliptic boundary-value problem on the spectrum of the limiting problem. In [5] V. G. Maz'ya, S. A. Nazarov, and B. A. Plamenevskii obtained full asymptotic expansions of the first eigenvalues and the corresponding eigenfunctions of the classical boundary-value problems for the Laplace operator in two-dimensional and three-dimensional domains with small holes.

In [6] I. V. Kamotskii and S. A. Nazarov considered the case of the Neumann edge conditions imposed on the boundary of a domain with a small cavity for an elliptic operator in the linear elasticity theory. They constructed the full asymptotic expansions of solutions to perturbed boundary-value problems.

In this paper we study a singularly perturbed Dirichlet boundary-value problem for an elliptic operator of the linear elasticity theory in a bounded domain with a small cavity. The main result is the proof of the theorem about the convergence of eigenelements of the perturbed boundary-value problem to eigenelements of the corresponding limiting boundary-value problem, when the parameter ε which defines the diameter of the cavity tends to zero.

1. PROBLEM DEFINITION AND STATEMENT OF RESULTS

Let Ω be a bounded domain in $\mathbb{R}^n$, $n \ge 2$, with a smooth boundary $\Gamma := \partial\Omega$. Let $\mathbf{u}(x)$, $x \in \mathbb{R}^n$, stand for the vector function

$$\mathbf{u}(x) := (u_1(x), \dots, u_n(x)).$$

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