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Existence and Continuity with Respect to Parameter of Solutions to Stochastic Volterra Equations in a Plane

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Abstract—In this paper we study stochastic Volterra equations in a plane. These equations contain integrals with respect to fields of locally bounded variation and square-integrable strong martingales. We prove the existence and the uniqueness of solutions of such equations with locally integrable (in some measure) trajectories, assuming that the coefficients of equations possess the Lipschitz property with respect to the functional argument. We prove that a solution of a stochastic Volterra integral equation in a plane is continuous with respect to parameter.

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INTRODUCTION

Let $(\Omega, \mathcal{F}, \mathsf{P})$ be a complete probabilistic space; denote by $\mathbb{F} = (\mathcal{F}_z, z \in \mathsf{R}^2_+)$ a two-parameter family of σ-algebras satisfying the conditions: 1) if $x \leqslant z$, then $\mathcal{F}_x \subset \mathcal{F}_z \subset \mathcal{F}$; 2) $\mathcal{F}_0$ contains all elements $\mathcal{F}$ of null probability; 3) $\mathcal{F}_z = \bigcap\limits_{x>z} \mathcal{F}_x$ for any z; 4) for any x and z the σ-algebras $\mathcal{F}_x$ and $\mathcal{F}_z$ are conditionally independent with respect to $\mathcal{F}_{x\wedge z}$.

In what follows $\mathcal{T}$ and $\mathcal{P}$ denote, respectively, σ-algebras of $\mathbb{F}$-progressively measurable and $\mathbb{F}$-predictable subsets of the space $R^2_+ \times \Omega$ [1].

The symbol $\mathcal{A}$ stands for the space of $\mathcal{T}$-measurable right continuous random fields $(A(z), z \in R^2_+)$, which have no discontinuities of the second kind and satisfy the condition $A(z) = 0$ for $z \in R^2_0 = (\{0\} \times R_+) \cup (R_+ \times \{0\})$ and $\mathsf{E}\,\mathrm{Var}_{[0,z]} A < \infty$ for $z \in R^2_+$;

The symbol $\mathcal{M}^2_S$ stands for the space of square-integrable strong $\mathbb{F}$-martingales, i.e., the space of $\mathcal{T}$-measurable right continuous random fields $(M(z),\ z \in R^2_+)$ which have no discontinuities of the second kind and satisfy the conditions $M(z) = 0$ for $z \in R^2_0$, $\mathsf{E}M^2(z) < \infty$ and $\mathsf{E}\{M(]z, z'])|\mathcal{F}^*_z\}=0$ for all $z, z' \in R^2_+$, $z \leqslant z'$, where $\mathcal{F}^*_z = \Big(\bigvee\limits_{z_2 \geqslant 0} \mathcal{F}_z\Big) \bigvee \Big(\bigvee\limits_{z_1 \geqslant 0} \mathcal{F}_z\Big)$, $M(]z, z']) = M(z') - M(z_1, z'_2) - M(z'_1, z_2) + M(z)$.

Let λ be a locally finite measure on $(R^2_+, \mathcal{B}(R^2_+))$; let $\mathcal{L}(R^2_+)$ stand for the completion with respect to the measure λ of the σ-algebra of Borel sets from R^2_+; let X_λ denote the space of all $\mathcal{L}(R^2_+)|\mathcal{B}(R)$-measurable functions $g : R^2_+ \mapsto R$ such that $\|g\|_z = \Big(\int\limits_{[0,z[} g^2 d\lambda\Big)^{1/2} < \infty$ for each $z \in R^2_+$. For $g \in \mathsf{X}_\lambda$ and $x \in R^2_+$ we define $g_x \in \mathsf{X}_\lambda$ by the equality $g_x(u) = g(u)\mathrm{I}^{(u)}_{[0,x[}$.

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