ISSN 1066-369X, Russian Mathematics, 2017, Vol. 61, No. 7, pp. 64–72. © Allerton Press, Inc., 2017.
Original Russian Text © E.N. Khasanova, 2017, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2017, No. 7, pp. 74–83.

Univalent Conformal Mappings Onto Polygonal Domains With Countable Set of Vertices by Generalized Christoffel–Schwarz Integral

E. N. Khasanova[1*]

[1] *Kazan State University of Architecture and Civil Engineering*
ul. Zelyonaya 1, Kazan, 420043 Russia

Received January 28, 2016

Abstract—We propose a formula for the conformal mapping of the upper half-plane onto a polygonal domain, which generalizes the Schwarz–Christoffel equation. It is obtained by terms of partial solution to the Hilbert boundary-value problem with a countable set of singularity points of the coefficients including a turbulence of logarithmic type at the infinity point. We also prove the existence of closed and univalent mappings.

DOI: 10.3103/S1066369X1707009X

Keywords: *Schwarz–Christoffel equation, conformal mapping, Hilbert boundary-value problem, univalent function.*

1. Statement of the problem. The well-known Schwarz–Christoffel formula determines conformal mapping of canonical domain onto a given polygon. The authors of a number of publications consider conformal mappings of upper half-plane on polygonal domains with infinite sets of vertices. They generalize the Schwarz–Christoffel formula on this case. There are published papers of two types. The first one includes publications (see, e.g., [1–4]), where the polygonal domains are determined and the formulas of conformal mappings contain unknown pre-images of vertices (so-called accessory parameters). At that, the polygonal domain frequently satisfy additional geometric restrictions such as invariance with respect transfer or similarity. We refer to the second type works [5–8], where both the pre-images of angular points and the angles at unknown vertices are given. The studies of the second type were begun by M. A. Lavrentiev in [1] concerning mapping of half-plane onto polygon with given angles at unknown vertices.

The present paper belongs to the second direction. It continues researches of R. B. Salimov and P. L. Shabalin [5–7], and also E. N. Karabasheva and P. L. Shabalin [8]. The author obtains a structure formula for conformal mapping of upper half-plane such that its boundary consists of two spiral-like broken lines with common beginning. In order to build the mapping there is used a formula obtained in [7] for special solution of the Hilbert boundary-value problem with countable set of discontinuity points. As a result, we generalize the Schwarz–Christoffel integral for the case of infinite set of vertices of polygonal domain. The schlichtness of these mappings is not evident, and requires additional studies. Therefore, the main issue of the research is the question of existence of univalent solution to the problem. We investigate this question by means of the lemma 2 from [8]. We have to note that paper [7], which deals with mappings on polygonal domains with unbounded rotation of tangent, did not concern schlichtness.

We will build a conformal mapping of upper half-plane $E = \zeta : \zeta = \xi + i\eta,\ \eta \geq 0$, of the complex plane onto simply connected polygonal domain D_z. Its boundary $L_z = \partial D_z$ consists of two broken lines L_z^1 and L_z^2 with common starting point $A_0(0,0)$. The broken lines L_z^1 and L_z^2 have infinite sets of rectilinear segments. We enumerate vertices of L_z^1 sequentially from A_0, and denote them $A_1, A_2, \ldots$. Analogously, we enumerate vertices of L_z^2 beginning from A_0, and denote them $A_{-1}, A_{-2}, \ldots$. When

*E-mail: enkarabasheva@bk.ru.