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HYPERBOLIC FAMILIES OF PLANES IN THE FIVE-DIMENSIONAL PROJECTIVE SPACE $(L_2^3)_2$

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The present paper relates to the field of classical differential geometry of ruled manifolds in multidimensional projective spaces. We investigate a special class of two-parameter families of two-dimensional planes $(L_2)_2$ in the five-dimensional projective space P_5. We introduce a class of hyperbolic families of planes $(L_2^3)_2$ such that one of the stationary straight lines describes a non-general pseudocongruence of straight lines. We prove that to each pseudocongruence of straight lines one can invariantly put in correspondence three hyperbolic, three weakly hyperbolic, and three parabolic plane families.

No summation is assumed with respect to the indices $i, j, k = 1, 2, 3$. If these indices appear in the same expression, they do not take equal values.

1. *Families* $(L_2^3)_2$. In P_5 let us take a projective frame $\{A_\lambda\}$ with infinitesimal motion equations

$$dA_\lambda = \omega_\lambda^\beta A_\beta, \quad \text{where} \quad d\omega_\lambda^\beta = \omega_\lambda^\gamma \wedge \omega_\gamma^\beta, \quad \lambda, \beta, \gamma = \overline{1, 6}.$$

Let us consider a two-parameter hyperbolic-type family of planes $(L_2)_2$ with property that each plane has three linearly independent focuses. Let the plane $L_2 = (A_1 A_2 A_3)$ describe the family $(L_2)_2$. For the independent forms on this family we take ω_1^4, ω_3^6. As in [1], we refer the family $(L_2)_2$ to the first order frame, then

$$\omega_i^{3+j} = 0. \tag{1}$$

Let us recall the geometric sense of the first order frame constructed in [1]: any point A_i is a focus of L_2 with focal direction $\omega_i^{3+i} = 0$, and the point A_{3+i} lies in the focal 3-plane of the focus A_i.

Taking the prolongation of this system, we get

$$\omega_i^j = a_i^j \omega_j + c_i^j \omega_i, \quad \omega_{3+i}^{3+j} = b_i^j \omega_i - c_i^j \omega_j, \quad \omega_2 = p_1 \omega_1 + p_3 \omega_3, \tag{2}$$

where $\omega_i^{3+i} = \omega_i$. Hence, by exterior differentiation, we find

$$\Delta a_i^j \wedge \omega_j + \Delta c_i^j \wedge \omega_i = 0, \quad \Delta b_i^j \wedge \omega_i - \Delta c_i^j \wedge \omega_j = 0,$$
$$\Delta p_1 \wedge \omega_1 + \Delta p_3 \wedge \omega_3 = 0,$$

where

$$\Delta a_i^j = da_i^j + a_i^j(2\omega_j^j - \omega_i^i - \omega_{3+j}^{3+j}) - a_k^j \omega_i^k,$$
$$\Delta b_i^j = db_i^j + b_i^j(\omega_i^i - 2\omega_{3+i}^{3+i} + \omega_{3+j}^{3+j}) + b_i^k \omega_{3+k}^{3+j},$$
$$\Delta c_i^j = dc_i^j + c_i^j(\omega_j^j - \omega_{3+i}^{3+i}) + c_i^k c_k^j \omega_k + \omega_{3+i}^j,$$
$$\Delta p_1 = dp_1 + p_1(\omega_1^1 - \omega_4^4 - \omega_2^2 + \omega_5^5),$$
$$\Delta p_3 = dp_3 + p_3(\omega_3^3 - \omega_6^6 - \omega_2^2 + \omega_5^5).$$