Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 1, pp.1–8, 1998 *UDC 517.92*

ON A METHOD FOR CONSTRUCTION OF LYAPUNOV'S FUNCTIONS FOR NONLINEAR NONAUTONOMOUS SYSTEMS

A.Yu. Aleksandrov

In the present article we investigate conditions for asymptotic stability of solutions of nonlinear systems of differential equations with nonstationary perturbations.

1. Consider the system

$$\dot{x}_s = \sum_{j=1}^{n} a_{sj} x_j, \quad s = 1, \ldots, n, \tag{1}$$

where a_{sj} are constant coefficients, and the corresponding perturbed system

$$\dot{x}_s = \sum_{j=1}^{n} (a_{sj} + b_{sj}(t)) x_j, \quad s = 1, \ldots, n. \tag{2}$$

We shall assume that the functions $b_{sj}(t)$ are continuous and bounded for $t \geq 0$ together with the integrals

$$\int_0^t b_{sj}(\tau) d\tau. \tag{3}$$

It was proved in [1] that from the asymptotic stability of system (1) there may not follow the asymptotic stability of system (2). In other words, the perturbations of the form under consideration may violate asymptotic stability of linear systems.

Further, let us consider the nonlinear system

$$\dot{x}_s = f_s(\mathbf{X}) + \sum_{j=1}^{k} b_{sj}(t) h_j(\mathbf{X}), \quad s = 1, \ldots, n. \tag{4}$$

Here $f_s(\mathbf{X})$ and $h_j(\mathbf{X})$ are continuously differentiable homogeneous functions of orders μ and σ, respectively, $\mu = p/q$, $\mu > 1$, $\sigma = u/r$, $\sigma > 1$, where p, q, u, and r are natural, moreover, q and r are odd, and the functions $b_{sj}(t)$ satisfy the conditions mentioned above.

Let the zero solution of the non-perturbed system

$$\dot{x}_s = f_s(\mathbf{X}), \quad s = 1, \ldots, n, \tag{5}$$

be asymptotically stable. In the case where the order of perturbation exceeds the order of the functions $f_s(\mathbf{X})$, i. e., $\sigma > \mu$, the zero solution of system (4) is also asymptotically stable (see [2]).

Let us show that for nonlinear systems the asymptotic stability of the zero solution can be preserved as well for $\sigma \leq \mu$.

Partially supported by the grant no. JKF 100 of International Scientific Foundation and Russian Government.

©1998 by Allerton Press, Inc.
Authorization to photocopy individual items for internal or personal use, or the internal or personal use of specific clients, is granted by Allerton Press, Inc. for libraries and other users registered with the Copyright Clearance Center (CCC) Transactional Reporting Service, provided that the base fee of $ 50.00 per copy is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923.