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ON ONE TYPE OF STABILITY OF A VECTOR COMBINATORIAL PROBLEM WITH PARTIAL CRITERIA OF THE FORMS Σ-MINMAX AND Σ-MINMIN

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1. Introduction

When investigating various kinds of stability of optimization problem, one meets a problem of finding some conditions, under which the set of optimal solutions possesses a certain prescribed invariance property with respect to an external influence. The problem stability is mostly defined as one of continuity or semi-continuity properties (for instance, in the sense of Hausdorff or Berge) of the optimal mapping, i. e., the point-to-set (multivalued) mapping, which establishes the correspondence between each set of problem's parameters and the set of solutions to be found. In discrete optimization problems, lower semi-continuity in the sense of Hausdorff is converted into the property of keeping initial optimal solutions of a problem under arbitrary changes of its parameters within a "small" neighborhood of its initial data. This kind of stability is accepted to be called a quasi-stability. A vast literature is devoted to its investigations for various vector discrete optimization problems. In [1] (see also [2]–[4]), it was proved that the coincidence of the Pareto set and the Smale set (the set of strictly effective solutions) is a necessary and sufficient condition for this kind of stability in vector integer linear programming problems with a bounded set of feasible solutions. This is not possible to extend this result to vector combinatorial problems with partial criteria of forms MINSUM, MINMAX and MINMIN. It was shown in [5] that the above condition becomes only sufficient, but not necessary, for such problems in the case when the linear partial criteria (MINSUM) are absent. Both a necessary and sufficient condition of quasi-stability of a vector trajectorial problem with any combination of the above partial criteria was obtained in [6] (see Corollary 2 below).

Investigations of the quasi-stability property were carried out for vector combinatorial problems with the lexicographic optimality principle [7], [8], the majoritary optimality principle [9], and also, the Pareto one only for l_∞-extremal trajectorial problems [10]–[12], for problems of linear form minimization over the set of substitutions [13] and for vector integer linear programming problems [4], [14].

In the present paper, we obtain two both necessary and sufficient conditions of quasi-stability of vector combinatorial problems with generalized partial criteria, including, in particular, some well-known ones such as linear and "bottle neck" ones. We also obtain some known before results from [6], [15] as corollaries.

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