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Conditional Correctness of Boundary-Value Problem for a Composite Fourth-Order Differential Equation

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Abstract—In this paper we investigate the uniqueness and the conditional stability of a solution to ill-posed boundary-value problem for an equation of mixed-composite type. We give proofs of uniqueness and conditional stability of a solution on a correctness set.

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This paper is dedicated to the investigation of ill-posed boundary-value problem for a mixed-composite type equation. Well-posed boundary-value problems for such types of equations were considered by many authors (for example, [1, 2]).

At the present time, definitions of well-posed problems for partial differential equations of mixed-composite type become more and more complicated in connection with necessities of modeling and control of processes in thermal physics and continuum mechanics. Ill-posed problems for the first-order and second-order equations have been sufficiently well-investigated [3, 4]. In the present paper we consider ill-posed problems for a third-order heterogeneous partial differential equation. Such equations have many different applications, for example, they describe processes of heat propagation in inhomogeneous media, interaction of filtration flows, mass transfer near the aircraft surface and complex flows of a viscous fluid [5].

1. Problem definition. Let us consider the differential operator

$$L \equiv \left(\operatorname{sgn} x \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial}{\partial t} + \frac{\partial^2}{\partial x^2}\right).$$

We investigate the function $u(x,t) \in C^{4,3}_{x,t}(\Omega) \cap C^{3,2}_{x,t}(\overline{\Omega})$ in the domain $\Omega = \{-1 < x < 1, 0 < t < T, x \neq 0\}$, satisfying the equation

$$Lu(x,t) = g(x,t) \tag{1}$$

under initial conditions

$$u(x,0) = p(x), \;\; u_t(x,0) = q(x), \;\; u_{tt}(x,0) = r(x), \;\; -1 \leqslant x \leqslant 1, \tag{2}$$

boundary conditions

$$\begin{gathered} u(-1,t) = u(1,t) = 0, \\ u_{xx}(-1,t) = u_{xx}(1,t) = 0, \;\; 0 \leqslant t \leqslant T, \end{gathered} \tag{3}$$

and gluing conditions

$$\begin{aligned} u(-0,t) &= u(+0,t), & u_x(-0,t) &= u_x(+0,t), \\ u_{xx}(-0,t) &= u_{xx}(+0,t), & u_{xxx}(-0,t) &= u_{xxx}(+0,t). \end{aligned} \tag{4}$$

Here $p(x), q(x) \in W^4_2[-1;1]$, $r(x) \in W^2_2[-1;1]$, $g(x,t) \in W^1_{2,t}(\Omega)$ are given functions.

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