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Criterion for Complete Indeterminacy of Limiting Interpolation Problem of Stieltjes Type in Terms of Orthonormal Matrix-Functions

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Abstract—The main goal of the paper is to study the ordered sequences of generalized interpolation problems and the limiting interpolation problem in the Stieltjes class by means of the orthonormal matrix-functions. We obtain explicit formulas for the orthonormal matrix-functions. The main result of the paper is a criterion for the complete indeterminacy of limiting interpolation problem in the Stieltjes class. General constructions are illustrated by examples of the Stieltjes moment problem and the Nevanlinna–Pick problem in the Stieltjes class.

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1. INTRODUCTION

A number of criteria for indeterminacy of interpolation problems with infinite sets of interpolation points is obtained in papers [1–7] in terms of Stieltjes parameters, orthonormal polynomials, Shur parameters, Weyl's nested circles and other truncated problems. The investigation was continued and generalized in different directions in the works of many authors. Let us mention a set of important for us papers [8–11], which are performed by means of V. P. Potapov's approach to solving interpolation problems for Nevanlinna's matrix-functions (MF). Analogous results for Stieltjes' MF are obtained in papers [12–15].

In the present paper we continue the investigation of ordered sequences of generalized interpolation problems and limiting interpolation problems for Stieltjes' MF (see [14, 15]). We show that an ordered sequences of generalized interpolation problems for Stieltjes' MF is related with two sequences of MF, which are orthonormal with respect to non-negative matrix measures on the positive half of the axis. Depending on the structure of truncated problems, the orthonormal MF can be either entire or meromorphic. They generalize orthonormal polynomials and rational functions. We obtain a criterion for complete indeterminacy of a limiting interpolation problem in terms of convergence of two series of orthonormal MF. This result is an analog of Hamburger's criterion for indeterminacy of the problem of moments. As the examples, we obtain criteria for complete indeterminacy of the Nevanlinna–Pick for Stieltjes' MF and of the matrix Stieltjes' problem of moments. Note that the criterion for complete indeterminacy of the matrix Stieltjes' problem of moments from Example 2 is new even for the classical Stieltjes problem of moments.

2. MAIN DEFINITIONS AND NOTATION

Let us introduce main definitions and formulate without proofs necessary information on the limiting interpolation problem (see the detailed description in [14]). We state and solve the problems in terms of analytical operator-functions (OF). Let $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$, $\mathbb{R}_- = \{x \in \mathbb{R} : x < 0\}$, $\mathbb{C}_+ = \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$, $\mathbb{C}_- = \{z \in \mathbb{C} : \operatorname{Im} z < 0\}$, $\mathbb{C}_\pm = \mathbb{C}_+ \cup \mathbb{C}_-$. We denote by $\mathcal{G}_1$, $\mathcal{G}_2$ separable Hilbert spaces, and by $\mathcal{H}$ finite-dimensional Hilbert spaces, $\{\mathcal{G}_1, \mathcal{G}_2\}$ stands for set of all bounded linear operators acting

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