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THE LINEAR CONVERGENCE RATE OF THE METHODS WITH ITERATIVE PROXIMAL REGULARIZATION

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1. Introduction

It is well known that solving a convex programming problem by a method of iterative proximal regularization, in a general case, one is not able to estimate the convergence rate. Taking into account special properties of the minimizing functional and an admissible set in certain classes of problems, one can estimate the convergence rate of the mentioned iterative method. In paper ([1], p. 885), assuming that a solution is unique, one establishes the linear convergence rate of the method of iterative proximal regularization. In [2] one justifies the linear convergence rate without an assumption on the uniqueness of the solution, but for a finite-dimensional case, with the help of the so-called quality functions ([2], p. 238). In this paper, we consider a certain abstract scheme which allows us to establish the linear convergence rate of the method with proximal regularization, solving semicoercive variational inequalities in mechanics; we make no assumptions on the uniqueness of a solution. The content of the paper is close to [3], but, as distinct from the latter, we perform the regularization in the norm of a weaker space. In addition, we consider in detail an application to a solution of a semicoercive contact problem in the elasticity theory.

2. An abstract scheme

Let V, H be Hilbert spaces; in addition, assume that $H \subset V$, H_1 is a finite-dimensional subspace in H, $Q_1 : H \to H_1$ is an orthoprojector, $Q_2 = I - Q_1$, where I is the identity operator, $H_2 = Q_2 H$.

Further, assume that a functional $\gamma : H \to R$ is defined such that $\gamma(v) = \gamma_1(Q_1 v) + \gamma_2(Q_2 v)$, where $\gamma_1 : H_1 \to R$ is a convex functional, and $\gamma_2 : H_2 \to R$ is a strongly convex functional (with the constant $\delta > 0$), i.e.,

$$\gamma_2(\lambda v_1 + (1-\lambda)v_2) \le \lambda\gamma_2(v_1) + (1-\lambda)\gamma_2(v_2) - \delta\lambda(1-\lambda)\|v_1 - v_2\|_H^2, \quad 0 \le \lambda \le 1, \quad \forall v_1, v_2 \in H_2.$$

Denote by $(\cdot,\cdot)$ and $\langle\cdot,\cdot\rangle$ the scalar products in the spaces H and V, correspondingly. Let us introduce in H a new scalar product

$$((u,v)) \equiv \langle u,v\rangle + \delta(Q_2 u, Q_2 v) \tag{1}$$

and assume that the corresponding norm $|||\cdot|||_H = \sqrt{((\cdot))}$ is equivalent to that $\|\cdot\|_H$.

Consider the extremal problem

$$\text{find } \min_{v \in G} \gamma(v), \tag{2}$$