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The Inverse Problem for the Lavrent'ev–Bitsadze Equation Connected with the Search of Elements in the Right-Hand Side

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Abstract—We consider an equation of mixed elliptic-hyperbolic type, whose right-hand side represents a product of two one-dimensional functions. We establish a criterion for the unique solvability of this equation and construct its solution as a sum of series on the set of its eigenfunctions. Under certain constraints imposed on the ratio of the rectangle sides, on boundary functions, and on known multipliers in the right-hand side of the equation, we obtain estimates separating small denominators that appear in coefficients of constructed series from zero.

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INTRODUCTION

Consider the following mixed-type elliptic-hyperbolic equation with an unknown right-hand side:

$$Lu \equiv u_{xx} + \operatorname{sgn} y \cdot u_{yy} = F(x,y) = \begin{cases} f(x)g_1(y), & y > 0; \\ f(x)g_2(y), & y < 0, \end{cases} \tag{1}$$

in a rectangular domain $D = \{(x,y) \mid 0 < x < l, -\alpha < y < \beta\}$, where α and β are given positive real-valued constants, and $g_1(y)$ and $g_2(y)$ are given functions.

The inverse problem. Find in the domain D functions $u(x,y)$ and $f(x)$ such that

$$u \in C^1(\overline{D}) \cap C^2(D_- \cup D_+), \;\; f(x) \in C(0,l) \cap L_2[0,l]; \tag{2}$$

$$Lu = F(x,y), (x,y) \in D_- \cup D_+; \tag{3}$$

$$u(0,y) = u(l,y) = 0, -\alpha \le y \le \beta; \tag{4}$$

$$u(x,\beta) = \varphi(x), u(x,-\alpha) = \psi(x), \;\; 0 \le x \le l; \tag{5}$$

$$u_y(x,-\alpha) = g(x), 0 \le x \le l, \tag{6}$$

where $\varphi(x)$, $\psi(x)$, and $g(x)$ are given sufficiently smooth functions, while $D_+ = D \cap \{y > 0\}$ and $D_- = D \cap \{y < 0\}$.

One can obtain boundary-value problems for mixed-type equations by transforming problems of the theory of transonic flows, direct and inverse problems connected with critical flows in Laval nozzles, and the theory of supersonic flows with local subsonic domains ([1], P. 303; [2–4]). The following equations are also referred to the mixed elliptic-hyperbolic type: plastic equilibrium equations under plane stress conditions, equations of a water flow in an open channel, where the flow rate is higher than

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