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NECESSARY CONDITIONS FOR THE NEGATIVITY OF GREEN'S FUNCTION OF TWO-POINT PROBLEM

Ye.I. Bravyĭ

1. Introduction

The question: What are the conditions under which a solution x of the boundary value problem

$$(\mathcal{L}x)(t) = f(t), \quad t \in [a,b]; \quad x(a) = 0, \quad x(b) = 0 \tag{1}$$

depends monotonely on the function f, attracted attention of many mathematicians and was considered for both ordinary differential operators $\mathcal{L}$ (see, e, g., [1], [2]), and functional-differential operators. We will consider a linear singular functional-differential second order operator $\mathcal{L}$. If in this case problem (1) has the unique solution $x = Gf$ for any right side of equation f, then the linear operator G is called Green's operator of the boundary value problem (1). The question about a monotone dependence of the solution of problem (1) on the right side of the equation reduces to the conditions for the monotonicity of Green's operator G of the boundary value problem.

A linear operator $A : X \to Y$, where both X and Y are semiordered spaces, is said to be *isotone* (*antitone*) and denoted by $A \geqslant 0$ ($A \leqslant 0$) if $Ax_2 \geqslant Ax_1$ ($Ax_2 \leqslant Ax_1$, respectively) for all $x_1 \leqslant x_2$, $x_1, x_2 \in X$.

We introduce the following notation: R^1 stands for the space of real numbers; $C[a,b]$ is the space of continuous functions $x : [a,b] \to R^1$ with the norm $\|x\|_C = \max\limits_{t\in[a,b]} |x(t)|$; $L[a,b]$ means the space of equivalence classes of integrable functions $z : [a,b] \to R^1$ with the norm $\|z\|_L = \int\limits_a^b |z(t)|\,dt$; χ_e stands for the characteristic function of the set $e \subset R^1$; $\rho(K)$ is the spectral radius of the operator K; by I we mean the identity operator in the corresponding space.

In the spaces $C[a,b]$ and $L[a,b]$ we will use the natural semiorder.

For linear ordinary differential second order equations the antitone property of Green's operator of problem (1) is equivalent to the presence of a positive solution of the homogeneous equation $\mathcal{L}x = 0$ (see, e. g., [3], [4]). In this case, Green's operator will be strictly antitone (see Definition 1.2 in what follows). Necessary conditions for the antitone property of Green's operator of problem (1) for the functional-differential equations were rather poorly studied: If the operator $\mathcal{L}$ has the form

$$\mathcal{L}x \stackrel{\text{def}}{=} \pi\ddot{x} - Tx,$$

where $T : C[a,b] \to L[a,b]$, $T \leqslant 0$ and $\pi(t) = (t-a)^{\mu_1}(b-t)^{\mu_2}$, $0 \leqslant \mu_1, \mu_2 \leqslant 1$, then the necessary and sufficient conditions for the antitone property of Green's operator G is the inequality

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