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LETTER TO EDITOR

Dear Editor,

In my article *"On ε-kernel of bounded set in a special metric space"* published in your journal "Russian Mathematics. Iz. VUZ", no. 9, pp. 32–35, 2000, Lemma 2 was formulated in an excessively general way. One can easily construct counterexamples, so Lemma 2 cannot be used to prove Corollary. Theorem and Corollary can be proved if we assume that (X,ρ) is a complete metric space which satisfies the following conditions:

1) *For any x, y in X a unique point $\omega(x,y)\in X$ exists such that for each positive integer n $\rho(x,\omega(x,y))=\rho(y,\omega(x,y))=\rho(x,y)/2$.*
2) *For any p, x, y in X, $2\rho(\omega(p,x),\omega(p,y))\le\rho(x,y)$.*
3) *For each $r>0$, and for any bounded sequences (p_n), (x_n), (y_n) in X such that $\rho(p_n,x_n)\le r$, $\rho(p_n,y_n)\le r$, $\lim\limits_{n\to\infty}\rho(p_n,\omega(x_n,y_n))=r$, we have $\lim\limits_{n\to\infty}\rho(x_n,y_n)=0$.*

Note that these conditions were formulated in [1], denoted there as A), B), C).

In what follows we will use the notation from the published paper. A complete metric space X satisfying 1) is a geodesic space where every two points $x,y\in X$ can be joined by a unique geodesic segment $[x,y]$ with endpoints x, y (see [2]). Condition 2) is the global condition of curvature nonpositiveness in Busemann sense (see [3], p. 304). The uniformly convex Banach spaces and the Lobachevskiĭ spaces (infinite-dimensional included) are simple examples of complete metric spaces which satisfy 1)–3). Now let us formulate the basic result in the precise form.

Theorem. *Let a complete metric space (X,ρ) satisfy 1)–3). For any sequence of nonempty bounded sets M_n, N_n such that $\delta(M_n,N_n)\to 0$, $n\to\infty$, and for each $\varepsilon>0$, the following assertions hold*

A. *$\delta(K_\varepsilon(M_n),K_\varepsilon(N_n))\to 0$, $n\to\infty$;*
B. *$\delta(\varepsilon(M_n),\varepsilon(N_n))\to 0$, $n\to\infty$;*
C. *If for any positive integer n the sets $k_\varepsilon(M_n)$, $k_\varepsilon(N_n)$ are nonempty, then $\delta(k_\varepsilon(M_n),k_\varepsilon(N_n))\to 0$, $n\to\infty$.*

The proof in the published paper (see p. 34-35) becomes correct if on p. 34 in lines 19 and 6 from the bottom, and on p. 34 in line 6 from the top we use Lemma below and, according to this Lemma, pass to the subsequence, and replace $c/3$ with a.

Lemma *Let a complete metric space (X,ρ) satisfy 1)–3). If for $r>0$, $c>0$, and for each positive integer l, $z_l\in X$, $M_l\in B[X]$, $W_l=\cap\{B[x,r]:x\in M_l\}\ne\emptyset$, $\rho(z_l,W_l)>c$, then $a>0$, a positive integer t_0, subsequences $(z_t)\subset(z_l)$, $(M_t)\subset(M_l)$ exist such that $\zeta_t=\sup\{\rho(z_t,B[x,r]):x\in M_t\}>a$ for each positive integer $t>t_0$.*

Proof. Suppose that the assertion of Lemma is false. Let $\zeta_l\to 0$, $l\to\infty$. Then one can find a positive number l_0 such that $\zeta_l<c/18$ for $l>l_0$.

For every positive integer l and each $x\in M_l$, we set $q_l(x)=[z_l,x]\cap S(x,r)$ for $z_l\in X\setminus B[x,r]$, $q_l(x)=z_l$ for $z_l\in B[x,r]$, where $S(x,r)$ is the ball of radius $r>0$ centered at x. Then, for each positive integer l we take $w_l\in W_l$ such that $\rho(z_l,w_l)<\rho(z_l,W_l)+c/6$. Then, for every $l>l_0$ and each $x\in M_l$, we obtain $\rho(\omega(w_l,q_l(x)),W_l)\ge\rho(q_l(x),W_l)-\rho(q_l(x),\omega(w_l,q_l(x)))\ge\rho(z_l,W_l)-\rho(z_l,q_l(x))-\rho(q_l(x),w_l)/2\ge\rho(z_l,W_l)-\zeta_l-\rho(q_l(x),z_l)/2-\rho(z_l,w_l)/2\ge\rho(z_l,W_l)-\zeta_l-\zeta_l/2-\rho(z_l,W_l)/2-c/12>c/3$.

Now the only possible cases are

Case 1. For every $l>l_0$ and each $x\in M_l$ there holds $\omega(w_l,q_l(x))\in W_l$. This contradicts the inequality $\rho(\omega(w_l,q_l(x)),W_l)>c/3$.