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The Complete System of Global Differential and Integral Invariants of a Curve in Euclidean Geometry

R. G. Aripov[1*] and D. Khadzhiev[2]**

[1]*National University of Uzbekistan, Vuzgorodok, Tashkent, 700174 Uzbekistan*
[2]*Karadeniz Technical University, 61080 Trabzon, Turkey*
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INTRODUCTION

Let $M(n)$ be the group of all isometries of the n-dimensional Euclidean space E_n and $SM(n)$ the group of all Euclidean motions in E_n (i. e., the subgroup in $M(n)$ generated by rotations and parallel translations in E_n). The Frenet–Serret equations for a curve in E_n give the curvature functions $k_1(s),\dots,k_{n-1}(s)$ of the curve ([1], P. 158–160; [2], P. 140–149). The curvatures $k_1(s),\dots,k_{n-2}(s)$ are $M(n)$-invariant. However, the curvature $k_{n-1}(s)$ is $SM(n)$-invariant but not $M(n)$-invariant. For example, the torsion of a curve in E_3 is an $SM(3)$-invariant but not $M(3)$-invariant function. Therefore, the system of curvatures $k_1(s),\dots,k_{n-1}(s)$ gives the solution to the problem of G-equivalence of curves only for the group $G = SM(n)$ ([1], P. 61–64). In addition, the method of orthogonal frame gives only conditions of local G-equivalence of curves.

For the classical theory of curves in the Euclidean space, we refer to [1] (P. 158–160), [2] (P. 140–149), [3] (P. 13–15), [4] (P. 185–194), and [5] (P. 61–64). Differential invariants, invariant parameterizations, and global properties of curves and paths in E_n are considered in papers [5–16] and books [2, 17].

This paper is devoted to the study of global G-equivalence of curves for the groups $G = M(n)$ and $G = SM(n)$.

The case of curves in equiaffine and centroaffine geometries is considered in the papers by D. Khadzhiev and O. Peksen [18, 19].

All paths and curves under consideration are assumed to be infinitely differentiable.

1. THE EUCLIDEAN TYPE AND AN INVARIANT PARAMETERIZATION OF A CURVE

Let R be a field of real numbers and $I = (a, b)$ the open interval in R. We realize the space E_n as the n-dimensional vector space R^n with scalar product $\langle x, y\rangle = \sum_{k=1}^{n} x_k y_k$ of vectors $x = (x_1,\dots,x_n)$ and $y = (y_1,\dots,y_n)$.

Definition 1. A C^∞-mapping $x : I \to R^n$ is called an I-path (path) in R^n.

Definition 2 ([18]). An I_1-path $x(t)$ and an I_2-path $y(r)$ in R^n are called D-equivalent if a C^∞-diffeomorphism $\varphi : I_2 \to I_1$ exists such that $\varphi'(r) > 0$ and $y(r) = x(\varphi(r))$ for all $r \in I_2$. A class of D-equivalent paths in R^n is called a curve in R^n (see [17], P. 9). A path $x \in \alpha$ is called a parameterization of a curve α.

*E-mail: arrustam@yandex.ru.
**E-mail: haciyev@ktu.edu.tr.