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ESTIMATES FOR MIXED NORMS OF THE SUMS OF DOUBLE TRIGONOMETRIC SERIES WITH MULTIPLY MONOTONOUS COEFFICIENTS

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In [1] by the same authors there were established certain properties of the sums of double trigonometric series in sines and cosines with multiply monotonous coefficients. The present article contains estimates of the sums of such series in the spaces $L_{\bar{p}}$ with a mixed norm.

1. Consider the series of the form

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} a_{m,n}\cos mx\cos ny, \quad \text{(I)} \qquad \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} a_{m,n}\cos mx\sin ny, \quad \text{(II)}$$

$$\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} a_{m,n}\sin mx\cos ny, \quad \text{(III)} \qquad \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{m,n}\sin mx\cos ny, \quad \text{(VI)}$$

where we set for brevity $\cos 0x = \cos 0y = \frac{1}{2}$. Assume that the coefficients of these series satisfy the following condition:

$$\begin{gathered} a_{m,n} \to 0 \text{ as } m \to \infty \text{ and for any fixed } n \\ \text{and as } n \to \infty \text{ and for any fixed } m. \end{gathered} \tag{1}$$

For integer numbers $k_1 \geq 0$ and $k_2 \geq 0$, write

$$\Delta_{k_1,k_2} a_{m,n} = \sum_{j=0}^{k_1}(-1)^i C_{k_1}^i \sum_{j=0}^{k_2}(-1)^j C_{k_2}^j a_{m+i,n+j}.$$

Let $B_0^1(X) = \frac{1}{2}$,

$$B_n^1(x) = \frac{1}{2} + \cos x + \cdots + \cos nx \qquad \text{for } n \geq 1,$$

$$B_n^k(x) = \sum_{\nu=0}^{n} B_\nu^{k-1}(x) \qquad \text{for } k = 2,3,\ldots \text{ and } n \geq 0;$$

$$\overline{B}_n^1(x) = \sin x + \cdots + \sin nx \qquad \text{for } n \geq 1,$$

$$\overline{B}_n^k(x) = \sum_{\nu=1}^{n} \overline{B}_\nu^{k-1}(x) \qquad \text{for } k = 2,3,\ldots \text{ and } n \geq 1.$$

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