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The Asymptotic Behavior of Solutions of a Certain Nonlinear Volterra Equation

Z. B. Tsalyuk[1*] **and M. V. Tsalyuk**[2**]

[1]*Kuban State University, ul. Stavropol'skaya 149, Krasnodar, 350040 Russia*
[2]*Krasnodar Regional Institute of Supplementary Professional Pedagogical Education, ul. Sormovskaya 167, Krasnodar, 350080 Russia*
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Abstract—We study the asymptotic behavior of bounded and unbounded solutions to the Volterra–Hammerstein equation. We obtain conditions for the admissibility of a pair of spaces consisting of the sum of a quasipolynomial and the Taylor expansion at infinity.

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The asymptotic behavior of solutions of linear Volterra equations is being investigated rather intensively (see, e.g., [1–6]). Here we study one of nonlinear equations, namely, the Volterra–Hammerstein one.

Let K be a locally summable real function. Denote

$$K * x(t) = \int_0^t K(t-s)x(s)\,ds, \quad \widehat{K}(z) = \int_0^\infty e^{-zt}K(t)\,dt.$$

Let the symbol $P(t)$ stand for various polynomials (we may indicate their degrees or not). Define the set

$$A(\lambda_1, \dots, \lambda_q, m) = \left\{x : x(t) = \sum_{k=1}^{q} P_k(t)e^{\lambda_k t} + \sum_{k=0}^{m} \frac{a_k}{(t+1)^k} + o\left(\frac{1}{(t+1)^m}\right) \text{ as } t \to \infty\right\}.$$

Let $\mathbf{BC} = \mathbf{BC}[0, \infty)$ be the space of continuous bounded functions,

$$\mathbf{C} = \{f \in \mathbf{BC} : \lim_{t\to\infty} f(t) = 0\}.$$

Consider the equation

$$x(t) = K * \big[x(t) + \varphi\big(t, x(t)\big)\big] + f(t), \tag{1}$$

where

$$K(t) = K_0(t) + \sum_{l=1}^{k} P_l(t)e^{\mu_l t} + \sum_{j=1}^{m} U_j * e^{i\nu_j t}P_j(t),$$

$\operatorname{Re}\mu_l \geq 0$, $e^{\beta t}K_0$, $e^{\beta t}U_j \in \mathbf{L}_1[0, \infty)$ with some $\beta > 0$.

As is known ([1], P. 40), the main terms of the asymptotics of the resolvent R of the kernel K depend on roots of the equation

$$1 - \widehat{K}(z) = 0$$

*E-mail: dif@math.kubsu.ru.
**E-mail: mv@kkidppo.ru.