*Russian Mathematics (Iz. VUZ)* *Izvestiya VUZ. Matematika*
*Vol. 45, No. 1, pp.46–51, 2001* *UDC 517.512.6*

# ESTIMATION OF THE ERROR OF TRIGONOMETRIC INTERPOLATION OF FUNCTIONS OF $m$-HARMONIC BOUNDED VARIATION

A.A. Kel'zon

In this article we deduce a bound for the deviation of the trigonometric interpolational Lagrange polynomial with equidistant points from the interpolated function under the assumption that the latter is continuous everywhere and its $m$-harmonic variation on a period is finite. The obtained bound yields the uniform convergence of the corresponding interpolational process for functions of class mentioned above.

Let us introduce the necessary notation. Let $m$ stand for a positive integer number, and $f$ for a $2\pi$-periodic function defined on the real axis. We denote by $V_{m,H}(f;a,b)$ an $m$-harmonic variation of the function $f$ on the segment $[a,b]$, i. e., (see [1])

$$V_{m,H}(f;a,b)=\sup\sum_{n=1}^{\infty}\frac{|\triangle^m f(I_n)|}{n}, \tag{1}$$

where

$$\triangle^m f(I_n)\equiv\triangle^m f(a_n,b_n)=\sum_{\nu=0}^{m}(-1)^{m-\nu}C_m^{\nu}f(a_n+\nu h_n),$$
$$h_n=(b_n-a_n)/m,$$

and the least upper bound in (1) is taken over all possible sequences $\{I_n\}$ of nonoverlapping intervals $I_n=(a_n,b_n)$ situated in $[a,b]$.

We denote by $\mathrm{CHBV}_m$ the class of continuous $2\pi$-periodic functions $f$ such that $V_{m,H}(f;0,2\pi)<\infty$.

Note that, for $m=1$, the corresponding class was introduced in [2]. The uniform convergence of the trigonometric Fourier series for functions of the class $\mathrm{CHBV}_1$ was proved in the same paper.

In what follows $\xi$ means an arbitrary real value,

$$x_{k,n}\equiv x_{k,n}(\xi)=\xi+2k\pi/(2n+1),$$
$$t_{k,n}(x)=\frac{\sin[(n+1/2)(x-x_{k,n})]}{(2n+1)\sin[(x-x_{k,n})/2]}=\frac{(-1)^k\sin[(n+1/2)(x-\xi)]}{(2n+1)\sin[(x-x_{k,n})/2]}, \tag{2}$$
$$k=0,\pm1,\dots;\quad n=0,1,2,\dots$$

We denote by $L_n(f;x)\equiv L_{n,\xi}(f;x)$, $n=1,2,\dots$, the trigonometrical polynomial of order not exceeding $n$, which coincides with the function $f$ at the points $x_{k,n}$, $k=0,\pm1,\dots$ (the trigonometric interpolational Lagrange polynomial). As is known (see, e. g., [3], Chap. X, § 1, p. 10),

$$L_n(f;x)=\sum_{k=-n}^{n}f(x_{k,n})t_{k,n}(x).$$