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ON THE SOLVABILITY OF A GEOMETRICALLY NONLINEAR PROBLEM IN THE THEORY OF SLANTING SHELLS

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The present article is devoted to the investigation of solvability of a geometrically nonlinear problem of theory of slating shells, which consists of determination of stressed-deformed state of free shells, which are not subject to any geometrical boundary conditions. The necessity of the study of problems for shells of that kind was noted by I.I. Vorovich in [1]. For investigation of these problems we suggest a method based on the solving a problem in deformations (see [2]).

1. In this item we shall introduce formulas for the displacement vector via components of deformation. To this end we shall use relations for components of a finite deformation, which were obtained by I.I. Vorovich (see [1]) relying on the Kirchhoff–Love hypotheses

$$\begin{aligned} &\varepsilon^0_{jj} = w_{j\alpha^j} - B_{jj}w - G^k_{jj}w_k + \tfrac{1}{2}w^2_{\alpha^j}, \quad j=1,2,\\ &\varepsilon^0_{12} = \tfrac{1}{2}(w_{1\alpha^2} + w_{2\alpha^1}) - B_{12}w - G^k_{12}w_k + \tfrac{1}{2}w_{\alpha^1}w_{\alpha^2},\\ &\varepsilon^1_{ij} = -w_{\alpha^i\alpha^j} + G^k_{ij}w_{\alpha^k}, \quad i\le j, \ \ i,j=1,2, \end{aligned} \tag{1}$$

where ε^0_{ij}, ε^1_{ij} are the components of tangential and bending deformations of the middle surface S_0; w_i, w are the tangential and normal displacements of the points of S_0; B_{ij} are components of the tensor of curvature of S_0; G^k_{ij} are the Christoffel symbols of second genus; α^1, α^2 are considered as Cartesian coordinates on a plane, which vary in a certain flat domain Ω.

We shall resolve system (1) with respect to w_i, w, assuming that ε^0_{ij}, ε^1_{ij} are known functions. To this end we pass from (1) to the system

$$w_{1\alpha^1} - w_{2\alpha^2} + (B_{22}-B_{11})w + (G^k_{22}-G^k_{11})w_k + \tfrac{1}{2}(w^2_{\alpha^1} - w^2_{\alpha^2}) = \varepsilon^0_{11} - \varepsilon^0_{22}, \tag{2}$$

$$\begin{aligned} w_{1\alpha^2} + w_{2\alpha^1} - 2B_{12}w - 2G^k_{12}w_k + w_{\alpha^1}w_{\alpha^2} &= 2\varepsilon^0_{12},\\ -w_{\alpha^1\alpha^1} - w_{\alpha^2\alpha^2} + (G^k_{11}+G^k_{22})w_{\alpha^k} &= \varepsilon^1_{11} + \varepsilon^1_{22}. \end{aligned} \tag{3}$$

We start solving system (2)–(3) from solving equation (3). We introduce notation: $a = -(G^1_{11} + G^1_{22})$, $b = G^2_{11} + G^2_{22}$, $\varepsilon_3 = \varepsilon^1_{11} + \varepsilon^1_{22}$, $u = -w_{\alpha^1}$, and $v = w_{\alpha^2}$. Then equation (3) will be equivalent to the system

$$\begin{aligned} u_{\alpha^1} - v_{\alpha^2} + au + bv &= \varepsilon_3,\\ u_{\alpha^2} + v_{\alpha^1} &= 0, \end{aligned}$$

which can be represented with the help of a complex function $W(z) = u(\alpha^1,\alpha^2) + iv(\alpha^1,\alpha^2)$, $z = \alpha^1 + i\alpha^2$ in the form

$$W_{\overline{z}} + AW + B\overline{W} = F, \tag{4}$$

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