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ON THE ASYMPTOTIC PROXIMITY OF THE SOLUTIONS OF THE CAUCHY PROBLEM FOR FIRST-ORDER DIFFERENTIAL EQUATIONS IN A BANACH SPACE

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In a Banach space $\mathbb{E}$ for $t > 0$ we consider the following equation:

$$v'(t) = \sum_{i=1}^{n} a_i(t)\mathbb{A}_i v(t), \tag{1}$$

where $\mathbb{A}_i$ are commuting generators of strongly continuous cosine-functions $\mathbb{C}_i(t)$ (see [1]), $a_i(t) \geq 0$ are continuous and bounded real-valued functions for $t \in [0, \infty)$. We shall study the asymptotic behavior as $t \to \infty$ of the solution of (1), which satisfies the initial condition

$$v(0) = v_0. \tag{2}$$

By developing the results on stabilization of the solutions of the Cauchy problem for parabolic equations (reviewed in [2], [3]), we shall deduce necessary and sufficient conditions for the stabilization of the solution of (1), (2) in the case where $n = 1$ and $a_i(t) \equiv 1$. Methods from [3], along with integral representations of the solutions obtained in [4], enable us to establish a necessary and sufficient condition for the stabilization of the solution of the Cauchy problem for the equation

$$w'(t) = \sum_{i=1}^{n} \alpha_i \mathbb{A}_i w(t) \tag{3}$$

with constant coefficients α_i. Note that the operator $\sum\limits_{i=1}^{n} \alpha_i \mathbb{A}_i$, generally speaking, is no longer a generator of a cosine-function (see [1]).

The purpose of this article is to prove a theorem on the asymptotic proximity of the solutions of the Cauchy problem for equations (1) and (3).

We denote by $\underline{\mathbb{D}(\mathbb{A})}$ a set, on which all possible compositions of the operators $\mathbb{A}_i$ are defined, and assume that $\overline{\mathbb{D}(\mathbb{A})} = \mathbb{E}$.

A counterpart of theorem 1 from [3] for the problem (1), (2) is

Theorem 1. *Let $v_0 \in \mathbb{D}(\mathbb{A})$ and for $i = 1, \dots, n$ $b_i(t) = \int_0^t a_i(x)dx$. Then the function*

$$v(t) = \left(4^n \pi^n \prod_{i=1}^{n} b_i(t)\right)^{-1/2} \int_{\mathbb{R}_n} \exp\left(-1/4 \sum_{i=1}^{n} y_i^2/b_i(t)\right) \prod_{i=1}^{n} \mathbb{C}_i(y_i) v_0 dy \tag{4}$$

is the unique solution of the problem (1), (2).