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On the Existence of Deformations of the Lie Algebras of Series Z

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Abstract—The paper continues a series of investigations devoted to the description of filtered deformations of the exceptional Lie algebras over algebraically closed fields of characteristic $p = 3$. The author constructs a realization of filtered deformations of the series Z Lie algebras as subalgebras in the infinite-dimensional algebra $Z(E)$. It is proved that these deformations are not isomorphic to the respective graded algebras.

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Investigation of filtered deformations of the Lie algebras presents an interest in connection with the classification problem for simple finite-dimensional Lie algebras over algebraically closed fields. At this moment the classification of the Lie algebras in characteristic $p = 2, 3$ remains unknown. This is partly due to the appearance of a large number of so-called exceptional Lie algebras. We will call Lie algebras isomorphic neither to the classical Lie algebras nor to those of Cartan type as exceptional algebras. In particular, such are the Melikyan algebras for $p = 5$ ([1, 2]) as well as the Frank algebras and the algebras of series R, X, Y, Z in characteristic $p = 3$ (e.g., [2, 3]). Earlier it was shown in the papers [1], [4–6] that some of them, namely the Melikyan algebras, the Frank algebras, and the algebras of series R and Y, are rigid with respect to filtered deformations.

However, there exist exceptional Lie algebras admitting nontrivial filtered deformations. So in the paper [3] S. M. Skryabin constructed not only $\mathbb{Z}$-graded Lie algebras of series X but also a family of their filtered deformations depending on a volume form ω. Up to now the question concerning the description of filtered deformations of the series Z algebras has not been investigated. In the present paper this problem is partly solved. More precisely, filtered deformations of the series Z graded Lie algebras with the standard grading are constructed as subalgebras in the infinite-dimensional Lie algebra of the same series. It is proved that these algebras are not isomorphic to the respective graded algebras.

By a filtered deformation of a $\mathbb{Z}$-graded Lie algebra $L = \bigoplus_{i=-q}^{r} L_i$ we understand a filtered Lie algebra $\mathcal{L} = \mathcal{L}_{(-q)} \supset \mathcal{L}_{(-q+1)} \supset \cdots \supset \mathcal{L}_{(r)} \supset (0)$ such that its associated graded algebra $\operatorname{gr}\mathcal{L}$ is isomorphic to L.

1. A DESCRIPTION OF FILTERED DEFORMATIONS OF THE LIE ALGEBRAS OF SERIES Z

The graded exceptional Lie algebras of series Z defined only over fields of characteristic $p = 3$ were constructed by S. M. Skryabin in [3]. Below we give a geometric realization of these algebras, preceding it with a brief overview of relevant terminology and notation (for details refer to [3, 7]).

Let E be a finite-dimensional vector space, $\mathcal{F} : E = E_1 \supseteq E_2 \supseteq \cdots \supseteq E_r = (0)$ a flag in E. Having fixed a flag-compatible basis $\{x_1, x_2, \ldots, x_n\}$ in E, we can put into a one-to-one correspondence with this flag a vector of heights $\overline{m} = (m_1, \ldots, m_n)$ by the rule $m_i = \max\{k : x_i \in E_k\}$. Further on the notations $\mathcal{F}$ and $\overline{m}$ will be considered mutually interchangeable.

For a flag $\mathcal{F}$ one defines the algebra of divided powers $\mathcal{O}(\mathcal{F})$, the general Lie algebra of Cartan type $W(\mathcal{F})$, the complex of differential forms $\Omega(\mathcal{F})$. By $Z(\Omega)$ and $B(\Omega)$ we denote the subcomplexes of exact

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