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ELLIPTIC BOUNDARY VALUE PROBLEM WITH SUPERPOSITION OPERATOR IN THE BOUNDARY VALUE CONDITION. I

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1. Statement of problem. Formulation of basic assertions

1. *Notation* (in what follows all values are assumed to be real). A point of the space R^n $(n \geq 2)$ is denoted by $x = (x_1, \dots, x_n)$; $\langle x, y\rangle = \sum\limits_{i=1}^{n} x_i y_i$, where $x, y \in R^n$; $|x| = \sqrt{\langle x, x\rangle}$; D is a bounded domain in R^n with the boundary S, $\overline{D} = D \cup S$; $N(x) = N = (N_1, \dots, N_n)$ stands for the unitary vector of exterior with respect to D normal to S at the point x, $P(x^0, r_0) = \{x \in R^n : |x - x^0| \leq r_0\}$, where $r_0 = \mathrm{const} > 0$, x^0 is a point of R^n; $S(x^0, r_0) = S \cap P(x^0, r_0)$; σ is the one-to-one mapping of R^n to R^n; $\omega = \sigma S \cap S$; $\rho(x, \omega) = \inf\limits_{\xi \in \omega} |x - \xi|$.

In this article we use the Hölder spaces defined in [1] (p. 112). The inclusion $z \in C_\alpha(D)$ means, in particular, that the function $z(x)$ is bounded in $\overline{D}$ and continuous by Hölder in D with the indicator $\alpha \in (0, 1)$; the inclusion $z \in C_{2+\alpha}(D)$ means, in particular, that $z(x)$ is continuous in $\overline{D}$ and possesses in D the derivatives $z_{x_i x_j}$ $(i, j = \overline{1, n})$ continuous by Hölder with the indicator α.

We denote by I the operator of taking the trace on S of functions given in $\overline{D}$: $(Iz)(x) = z(x)$ for $x \in S$; by A the superposition operator, i.e., $(Az)(x) = z(\sigma x)$ for $x \in S$.

Let us define several linear normed spaces which will play the essential role in our constructions. Assume that a nonempty set $e \subset S$ and numbers $\mu \in (0, 1)$, $\beta > 0$, be given. We denote by $C^{\mu\beta}(S, e)$ the space whose elements are continuous on S functions $\varphi(x)$ such that

$$\sup_{x, \xi \in e} \frac{|\varphi(x) - \varphi(\xi)|}{|x - \xi|^\mu} + \sup_{x \in S \setminus e,\ \xi \in e} \frac{|\varphi(x) - \varphi(\xi)|}{|x - \xi|^{\mu\beta/2}} + \max_{x \in S} |\varphi(x)| \equiv \varkappa_{\mu\beta}(\varphi) < \infty.$$

In the capacity of the norm φ in this space we take $\varkappa_{\mu\beta}(\varphi)$ or any other norm equivalent to $\varkappa_{\mu\beta}$. In case $e = S$ the given space coincides with the usual Hölder space $C^\mu(S)$. We denote by $\overset{0}{C}{}^{\mu\beta}(S, e)$ the space $C^{\mu\beta}(S, e)$ consisting of functions vanishing on e.

We denote by $C^{\mu\beta}_{2+\alpha}(D, e)$ the space consisting of functions $z \in C_{2+\alpha}(D)$ for which $Iz, Az \in C^{\mu\beta}(S, e)$; the norm in this space is defined as the sum of corresponding norms of z, Iz, Az. $\overset{0}{C}{}^{\mu\beta}_{2+\alpha}(D, e)$ stands for the subspace of $C^{\mu\beta}_{2+\alpha}(D, e)$, which consists of those elements $z(x)$, which obey $Iz, Az \in \overset{0}{C}{}^{\mu\beta}(S, e)$.

2. *Statement of problem. Formulation of theorems.*

In this article we study the relation between the (classical) resolvability of the problem

$$\mathcal{L}u \equiv \sum_{i,j=1}^{n} a_{ij}(x) u_{x_i x_j} + \sum_{i=1}^{n} b_i(x) u_{x_i} + c(x) u = f(x), \quad x \in D, \tag{1}$$

$$Bu \equiv u(x) - u(\sigma x) = \psi(x), \quad x \in S, \tag{2}$$